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ON REPRESENTATION OF JAPANESE CANDLESTICKS BY ORDERED FUZZY NUMBERS

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Abstract: *The Japanese candlesticks' technique is one of the well-known methods of dynamic analysis of securities. Kacprzak et al. (2013) presented a proposal to describe Japanese candlesticks by ordered fuzzy numbers which are introduced by Kosiński and his cooperators. In view of some reservations made on the mathematical field towards the ordered fuzzy number term, in this work we again take on the subject of Japanese candles' modeling. Our study is coherent with cognitive paradigm of limitation the family of all ordered fuzzy number to the family of proper ordered fuzzy number. The main kinds of Japanese candles – Black Spinning and white spinning are considered. Moreover, there are considered: Doji Star, Long Legged Doji, Dragonfly Doji, Gravestone Doji and Four Price Doji. As a result of these considerations, we have rewritten the model of the Japanese candle described as ordered fuzzy number. In our model all Japanese candles are described by proper ordered fuzzy numbers.*

Key words: *Japanese candlestick, ordered fuzzy number, dynamic analysis of securities*

JEL codes: *C44, C02, G10*

1. Introduction

The ordered fuzzy numbers (OFNs) were introduced by Kosiński and his cooperators in the series of papers (Kosiński and Słysz, 1993; Kosiński et al., 2002a, 2002b; 2003; Kosiński, 2006) as an extension of the fuzzy numbers. Kosiński (2006) has shown that there exist improper OFNs which cannot represented by a pair of fuzzy number and orientation. We cannot to apply any knowledge of fuzzy sets to solve practical problems described by improper OFN Therefore our considerations with use improper OFN may be not fruitful.

Japanese candlesticks (Nison, 1997) are very useful tool supporting investors' decision on exchange market. Kacprzak et al. (2013) have described Japanese sticks by means of OFNs in this way that any white candle describes rise in market quotations and any black candle describes fall in market quotations. The Kacprzak's description has a significant disadvantage. There each black candle is described by improper OFN. So, the Kacprzak's description is not convenient for financial analysis with OFN use.

The above conclusion justifies remodelling Japanese candlesticks with OFN use. The main aim of this paper is to describe Japanese candlesticks by means of proper OFN.

2. The basic notions

An imprecise number is a family of values in which each considered value belongs to it in a varying degree. A commonly accepted model of imprecise number is the fuzzy number (FN), defined by Dubois and Prade (1979) as some kind of fuzzy subset in the real line $\mathbb{R}$. We restrict our considerations to the case of trapezoidal FN (TrFN) defined in the following way.

Definition 1: For any nondecreasing sequence $\{a, b, c, d\} \subset \mathbb{R}$, the trapezoidal fuzzy number $\overleftrightarrow{Tr}(a, b, c, d)$ is determined explicitly by its membership functions $\mu_{Tr}(\cdot\,|a, b, c, d) \in [0,1]^{\mathbb{R}}$ as follows:

$$\mu_{Tr}(x|a,b,c,d)=\begin{cases}0, & x\notin[a,d]=[d,a],\\ \frac{x-a}{b-a}, & x\in[a,b[\,=\,]b,a],\\ 1, & x\in[b,c]=[c,b],\\ \frac{x-d}{c-d}, & x\in\,]c,d]=[d,c[.\end{cases}\quad \square \tag{1}$$

The concept of ordered fuzzy numbers (OFN) was introduced intuitively by Kosiński and his co-writers in the series of papers (Kosiński and Słysz, 1993; Kosiński et al., 2002a; 2002b; 2003; Kosiński, 2006) as an extension of the concept of fuzzy numbers. The intuitive Kosiński's approach to the notion of OFN is very useful. We can say that any OFN $\overleftrightarrow{S}$ is explicitly determined by its membership relation $\mu_S \subset \mathbb{R}\times[0,1]$ where the value $\mu_S(x)\in[0,1]$ is interpreted as degree in which the real number x belongs to OFN $\overleftrightarrow{S}$.

We have restricted our considerations to the case of TrFN. For these reasons, below we present such definition of trapezoidal OFN which fully corresponds to the intuitive OFN definition by Kosiński.

Definition 2: For any sequence $\{a,b,c,d\}\subset\mathbb{R}$ Kosiński's trapezoidal ordered fuzzy number (K's-TrOFN) $\overleftrightarrow{Tr}(a,b,c,d)$ is defined explicitly by its membership relation $\mu_{Tr}(\cdot\,|a,b,c,d)\subset\mathbb{R}\times[0,1]$ given by (1). □

Any sequence $\{a,b,c,d\}\subset\mathbb{R}$ satisfies exactly one of the following conditions:

$$b<c\ \boldsymbol{or}\ (b=c\ \boldsymbol{and}\ a<d), \tag{2}$$

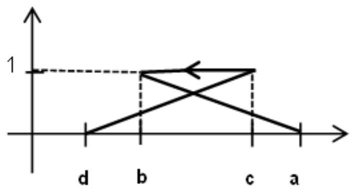
$$b>c\ \boldsymbol{or}\ (b=c\ \boldsymbol{and}\ a>d), \tag{3}$$

$$a=b=c=d. \tag{4}$$

If the condition (2) is fulfilled then the K's-TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ has positive orientation. Positive oriented K's-TrOFN is interpreted as such imprecise number, which may increase. If the condition (3) is fulfilled then the K's-TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ has fulfilment negative orientation. Negative oriented K's-TrOFN is interpreted as such imprecise number, which may decrease. If the condition (4) is fulfilled then the TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ describes $a\in\mathbb{R}$ which is not oriented. The monograph of Prokopowicz et al. (2017) is a competent source of information about the contemporary state of knowledge on OFN.

If the sequence $\{a,b,c,d\}\subset\mathbb{R}$ is not monotonic then the membership relation $\mu_{Tr}(\cdot\,|a,b,c,d)\subset\mathbb{R}\times[0,1]$ is not a function. Then this membership relation cannot be considered as membership function of any fuzzy set. Therefore, for any non-monotonic sequence $\{a,b,c,d\}\subset\mathbb{R}$ the K's-TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ is called improper OFN (Kosiński, 2006). Some example of improper K's-TrOFN is presented in figure 1.

Fig. 1 Membership relation of improper K's-TrOFN $\overleftrightarrow{Tr}(d,c,b,a)$



Source: Own study

In general, if OFN is determined by such membership relation which is not function then it is called improper OFN. On the other side, if OFN is determined by such membership relation that it is function then it is called proper OFN. Only in the case of proper OFN we can to apply all knowledge of fuzzy sets to solve practical problems described by OFN. In with way, our practical considerations will be more fruitful. Therefore, in our opinion, limiting the OFNs' family to the family of proper OFNs is useful and very necessary. Thus the cognitive paradigm of limiting the OFNs' family to the family of proper OFNs is well justified.

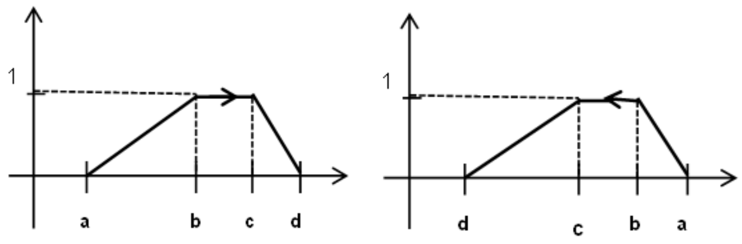
Only in the case of any monotonic sequence $\{a,b,c,d\}\subset\mathbb{R}$ the membership relation $\mu_{Tr}(\cdot\,|a,b,c,d)\subset\mathbb{R}\times[0,1]$ may be interpreted as membership function $\mu_{Tr}(\cdot\,|a,b,c,d)\in[0,1]^{\mathbb{R}}$. Thus we distinguish following kind of proper OFN.

Definition 3: For any monotonic sequence $\{a,b,c,d\}\subset\mathbb{R}$, the trapezoidal ordered fuzzy number (TrOFN) $\overleftrightarrow{Tr}(a,b,c,d)$ is determined explicitly by its membership functions $\mu_{Tr}(\cdot\,|a,b,c,d)\in[0,1]^{\mathbb{R}}$ given by (1). □

The graph of the positive oriented TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ membership function has an extra arrow denoting the positive orientation, which provides supplementary information. An example of such graph is presented in figure 2a. For the case, the graph of the TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ membership function has an extra arrow denoting the negative orientation. An example of such graph is presented in figure 2b.

Fig. 2 The membership function of TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ with: a) positive orientation, b) negative orientation

a) b)

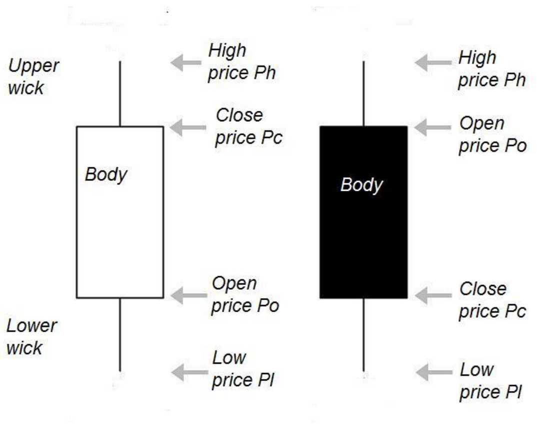


Source: Own elaboration

3. Japanese candlestick

A Japanese candlestick is a style of financial chart used to describe variability of financial assets exchange quotations. Candlestick charts have been developed in the 18th century by a Japanese rice trader Munehisa Homma (Morris, 2006)**.** The Japanese candlestick charting techniques were introduced to Western world by Nison (1997).

Fig. 3 Japanese candlesticks



Source: Own elaboration

For given time period $[0,T]$, the variability of financial assets exchange quotations is described by the function $Q:[0,T] \longrightarrow \mathbb{R}^+$. Each candlestick represents all four important pieces of information for fixed period of exchange quotations:

- the open price:

$$Po = Q(0), \tag{5}$$

- the close price:

$$Pc = Q(T), \tag{6}$$

- the high price:

$$Ph = \max\{Q(t): t \in [0,T]\}, \tag{7}$$

- the low price:

$$Pl = \min\{Q(t): t \in [0,T]\}. \tag{8}$$

The high price and the low price together are called extreme prices. In general, Japanese candlesticks are composed of:

- the body determined as a rectangle between the open price and the closed price,
- the upper wick determined as the line between the body and the high price,
- the lower wick determined as the line between the body and the low price.

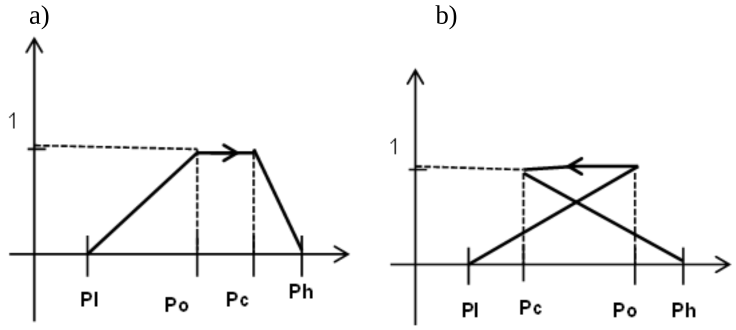
The body illustrates the opening and closing trades. If the open price is higher than the close price then the candlestick' body is black. If the open price is lower than the close price then the candlestick' body is white. A candlestick need not have either a body or a wick. The general case of white and black Japanese candlestick is presented in figure 3.

Some applications of Japanese candlesticks are presented for example in Detollenaere and Mazza (2014), Fock et al. (2005), Jasemi et al. (2011), Marshall et al. (2006), Takemori and Cihan (2009), and Lu et al. (2012, 2015).

4. The Kacprzak's representation of Japanese candlesticks

Kacprzak et al. (2013) have proposed the representation of Japanese candlesticks by K's-TrOFN $\overleftrightarrow{Tr}(Pl, Po, Pc, Ph)$. Then any white candlestick is represented by positive oriented TrOFN determined by the membership function presented in the Fig.4a. In this representation any black candlestick is described by such negative oriented K's-TrOFN which is improper. In the Fig.4b we can see the graph of a membership relation describing a black candle.

Fig. 4 Membership relation representing Japanese candlesticks: a) white candlestick, b) black candlestick



Source: Own elaboration

As we see, the Kacprzak's representation of Japanese candlesticks is not coherent with our cognitive paradigm on limitation OFNs' family to the family of proper OFNs. Thus, in the next section we will propose such Japanese candlesticks representation that any candlestick will be represented by proper TrOFN.

5. Our proposition of Japanese candlesticks representation

By modifying the candle model, we will first change the identification of extreme prices.

If the extreme price $Pb \in \{Pl, Ph\}$ is closer to the open price Po than to the close price Pc then the price Pb is called back price. In other words, the extreme price $Pb \in \{Pl, Ph\}$ is called back price iff it fulfills the inequality:

$$|Pb - Po| \leq |Pb - Pc|. \tag{9}$$

It means that the back price Pb is determined explicitly iff the condition:

$$Pc \neq Po \tag{10}$$

is fulfilled. In this way, for the white candle the back price Pb is equal to the low price Pl and for black candle, the back price Pb is equal to the high price Ph.

If the extreme price $Pf \in \{Pl, Ph\}$ is closer to the close price Pc than to the open price Po then the price Pf is called face price. In other words, the extreme price $Pb \in \{Pl, Ph\}$ is called face price iff it fulfills the inequality:

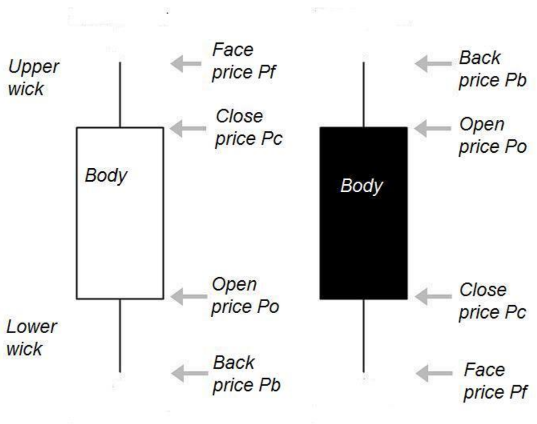
$$|Pf - Pc| \leq |Pf - Po|. \tag{11}$$

It means that the face price Pf is determined explicitly iff the condition (10) is fulfilled. In this way, for the white candle the face price Pf is equal to the high price Ph and for the black candle the face price Pf is equal to the low price Pl.

With this reinterpretation of extreme prices we have obtained such a modernized Japanese candle model in which the direction from the open price to close prise is equal to the direction from back price to face price. The general case of white and black Japanese candlestick is presented in figure 5.

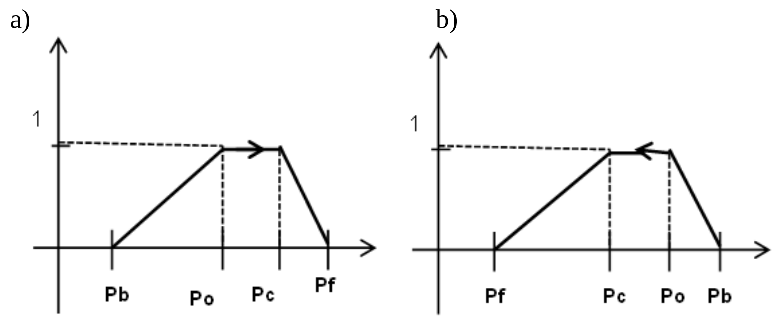
We propose to describe Japanese candlesticks by TrOFN $\overleftrightarrow{Tr}(Pb, Po, Pc, Pf)$. Then any white candlestick is represented by positive oriented TrOFN determined by the membership function presented in the Fig. 6a. In this representation any black candlestick is described by negative oriented TrOFN determined by the membership function presented in figure 6b.

Fig. 5 Japanese candlesticks – modernized model



Source: Own elaboration

Fig. 6 Membership function representing modernized Japanese candlesticks: a) white candlestick, b) black candlestick



Source: Own elaboration

If any Japanese candlestick does not fulfill the condition (10) then it does not have body. Such Japanese candlestick is called Doji. All main kinds of Doji are presented in figure 7.

For any Doji the back price and the face price are not determined explicitly. Thus the orientation of TrOFN representing any Doji cannot be defined as the direction from back price to the face price. Therefore, we propose to represent any Doji by TrOFN with orientation from the earlier extreme price to later extreme price.

Mentioned above prices are determined in the following way. At the beginning, for each observed quotation $\hat{Q} \in Q([0,T])$ we set the last moment of quotation $\tau(\hat{Q})$ defined by the identity:

$$\tau(\hat{Q}) = \max\{t \in [0,T]: Q(t) = \hat{Q}\}. \tag{12}$$

Let us consider now the case when:

$$\tau(Pl) \neq \tau(Ph). \tag{13}$$

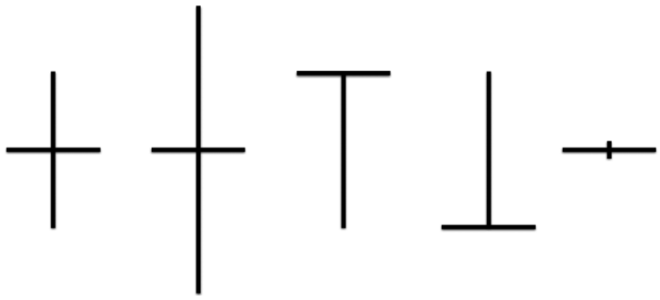
Then the earlier extreme price ePe and later extreme price lPe we define as follows:

$$ePe = \begin{cases} Pl, & \tau(Pl) < \tau(Ph), \\ Ph, & \tau(Pl) > \tau(Ph), \end{cases} \tag{14}$$

$$lPe = \begin{cases} Ph, & \tau(Pl) < \tau(Ph), \\ Pl, & \tau(Pl) > \tau(Ph). \end{cases} \tag{15}$$

Fig. 7 The main kinds of Doji:
a) Doji Star, b)Long Legged Doji, c)Dragonfly Doji, d) Gravestone Doji, e) Four Price Doji

a) b) c) d) e)

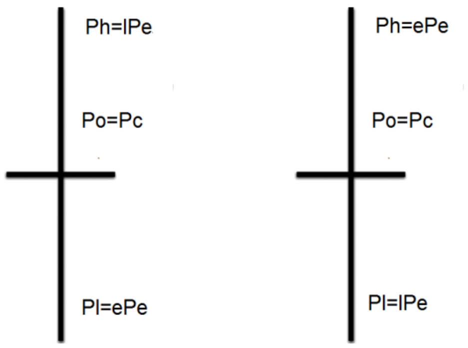


Source: Own elaboration

In this way we have interpreted extreme prices as earlier or later one. With this interpretation of extreme prices we have obtained such Doji model in which the orientation may be determined as direction from the earlier price to the later price. The general cases of Doji representation is shown in figures 8a and 8b.

Fig. 8 Doji – modernized model: a) positive oriented Doji, b) negative oriented Doji

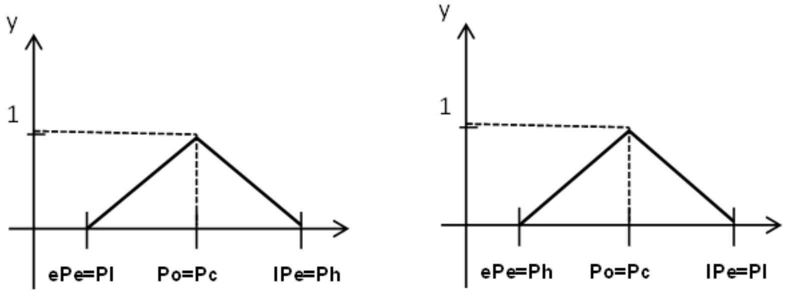
a) b)



Source: Own elaboration

We propose to describe Doji by TrOFN $\overleftrightarrow{Tr}(lPe, Pc, Pc, ePe)$. Then the Doji presented in the Fig. 8a is described by the positive oriented TrOFN $\overleftrightarrow{Tr}(Pl, Pc, Pc, Ph)$. The Doji presented in the Fig 8b is described by negative oriented TrOFN $\overleftrightarrow{Tr}(Ph, Pc, Pc, Pl)$. These representations of Doji are determined by their membership functions presented in figure 9.

Fig. 9 Membership function representing modernized Doji: a) positive oriented Doji, b) negative oriented Doji



Source: Own elaboration

We propose to describe Doji by TrOFN $\overleftrightarrow{Tr}(lPe, Pc, Pc, ePe)$. Then the Doji presented in the Fig. 8a is described by the positive oriented TrOFN $\overleftrightarrow{Tr}(Pl, Pc, Pc, Ph)$. The Doji presented in the Fig 8b is described by negative oriented TrOFN $\overleftrightarrow{Tr}(Ph, Pc, Pc, Pl)$. These representations of Doji are determined by their membership functions presented in the Fig. 9.

For any Dragonfly Doji we have:

$$\tau(Ph) = \tau(Pc) = T > \tau(Pl). \tag{16}$$

Thus, in agree with (14) and (15) any Dragonfly Doji is represented by positive oriented TrOFN $\overleftrightarrow{Tr}(Pl, Pc, Pc, Pc)$. This positive orientation is consistent with the common belief that any Dragonfly Doji could signal a potential bullish reversal of market quotations. The Dragonfly Doji representation is shown in the Fig. 10a.

For any Gravestone Doji we have:

$$\tau(Pl) = \tau(Pc) = T > \tau(Ph). \tag{17}$$

Thus, in agree with (14) and (15) any Gravestone Doji is represented by negative oriented TrOFN $\overleftrightarrow{Tr}(Ph, Pc, Pc, Pc)$. This negative orientation is consistent with the common belief that any Gravestone Doji could signal a potential bearish reversal of market quotations. The Gravestone Doji representation is shown in figure 10b.

In the condition (13) is not fulfilled, then we obtain:

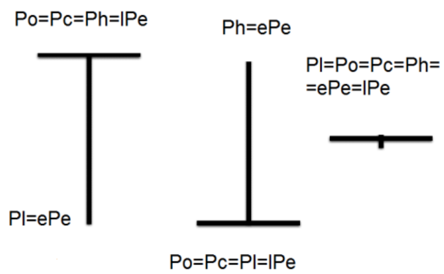
$$Pl = Pc = Pc = Ph. \tag{18}$$

This case corresponds to the Four Price Doji which is represented in agree with (14) and (15) by the close price:

$$\overleftrightarrow{Tr}(Pc, Pc, Pc, Pc) = Tr(Pc, Pc, Pc, Pc) = Pc. \tag{19}$$

The Gravestone Doji representation is shown in figure 10c.

Fig. 10 Doji – modernized model: a) Dragonfly Doji, b) Gravestone Doji, c) Four Price Doji
a) b) c)



Source: Own elaboration

It is very easy to check that all types of Japanese candles omitted in the above specification are only special cases of discussed candles.

4. Conclusions

In this paper we have shown that any kind of Japanese candlestick may be explicitly represented by proper trapezoidal ordered fuzzy number. Therefore we can apply the whole fuzzy sets theory to the analysis of Japanese candlesticks.

Our approach to fuzzy representation of Japanese candlestick differs to analogous approach presented in Marszałek and Burczyński (2017), where the memberships function of candles representation is defined as some density function.

On the other side, defined by us fuzzy representation of Japanese candles may be applied to fuzzy portfolio analysis as an example of imprecise present value.

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