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FORECASTING FROM AN ECONOMETRIC MODEL WITH FEEDBACK

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Abstract: *Prediction from a system of interdependent equations can be done in two ways. In the first procedure, equations of a reduced-form model are used. In the second proceeding, however, inference to the future is based on structural-form equations. Forecasts from reduced-form equations may, however, turn out to be divergent, especially in econometric micromodels.*
This paper presents a procedure of a prediction from a system of interdependent equations, based on a structural form, which can be called a reduced-chain from. The procedure consists in "breaking" of the feedback. The forecast obtained from a reduced-form equation will be used to construct an econometric forecast from the second structural-form equation.
Next, the forecast from the structural-form equation will be used to build a forecast equation, which the previous forecast was constructed from, based on the reduced-form equation. If the forecast obtained from the reduced form differs from the forecast obtained from the structural-form equation, then the proceeding is continued using the last forecast from the structural-form of the second equation. The prediction procedure is continued until a pair of convergent prognoses is obtained. The procedure will be illustrated with an empirical example based on empirical data obtained from an actual company. The empirical example will reveal the difference between the forecasts obtained from the structural form, in comparison with the forecasts from the reduced form of the econometric micromodel.

Key words: *econometric prediction, econometric forecast, predictor, feedback, system of interdependent equations.*

JEL codes: *C01, C32, C53*

1. Introduction

Prediction from a system of interdependent equations is not one of the issues often presented in economic literature. Interest in econometric macromodels, in the past, caused prediction from interdependent equation systems to be treated only referentially in literature. The systems of interdependent equations known in literature mainly are models of national economies of various countries.

Macromodels most commonly are based on data in the form of yearly time series, which are characterized by a "smooth" waveform. Econometric models based on quarterly data appear only exceptionally. In such cases, description accuracy of each equation usually is high, since cases of a convergence coefficient R^2 at a level above 0.95, often reaching the value of 0.99, are dominant. In such situation, issues of possible discrepancies in the forecasts obtained from the reduced form are not noticed when they are confronted with the prediction results from the structural-form equations of a model.

The purpose of this paper is to present an authorial method of prediction from a system of interdependent equations, which is based on empirical structural-form equations and designed mainly for econometric micromodels. The prediction procedure will be analogous to the so-called chain prediction, proper for a recursive model. The difference – in comparison with a prediction from a recursive model – lies in the need to use one of the empirical reduced-form equations to initiate a procedure of building a forecast series from subsequent empirical structural-form equations. In consequence, a proposition of a procedure of prediction from a system of interdependent equations, which can be described as a ***reduced-chain*** one emerges (Wiśniewski 2016a, pp. 43-45; Wiśniewski, 2017). The procedure can also be called a ***snail*** procedure. This contributes to the theory of constructing econometric prognoses under the circumstances indicated below. The proposed econometric prediction procedure has been illustrated by an empirical example, based on data from an actual existing medium-sized enterprise.

2. Methodology and Data

Predictions from a system of interdependent equations can be done in two ways. In the first proceeding, structural-form equations of the model are used, while in the second, inference into future is made based on reduced-form equations. These methods do not replace each other, especially when the system of equations is identifiable ambiguously.

Reduced-form equations can be used when existence of mutual causal links between stochastic interdependent variables is omitted in the considerations as well as for estimation of an effect of only one dependency of these variables. In particular, such a proceeding is suitable in the case of econometric macromodels constructed on the basis of yearly time series. The procedure is similar to that used in the case of simple equations. The value of the endogenic variables playing the role of the explanatory ones in the equation is then determined for the forecast period T using the same methods as for exogenous variables.

Prediction based on structural-form equations, taking into account only one side of the multilateral links between the interdependent variables, thus has the nature of inference into the future for short periods only (Pawłowski, 1973, pp. 259-265; Zeliaś, 1997, p. 20). Only in a very short period, abstraction from the other side of the interdependency between the variables can be made. In longer periods, the interdependencies between the endogenous variables play a significant role, while their omission can distort the essence and the results of the forecast research.

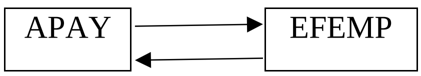
For this reason, the second method of inference into the future – based on reduced-form equations of a model – has greater practical significance, on the macroscale. In this method, a forecast can be treated as a conditional mathematical expectation, where predetermined variables occur in the condition. Prediction is made on the basis of each reduced-form equation separately. The procedure here is identical to that used in the case of a simple model, since the reduced-form has the nature of a simple model.

If the parameters of the reduced-form equations have been estimated directly, then the variations and the covariations of the structural parameter estimations of each equation of this form are known. It is easy then to determine the prediction variations for each equation. It is more difficult, however, when the reduced-form has been determined from an empirical structural form. It is worth noticing that, usually, the reduced-form equations, each of which contains all the predetermined variables, are characterized by occurrence of statistically insignificant explanatory variables. As a consequence, large, usually, average prediction errors occur, calculated from the reduced form. Therefore, the average prediction errors for the forecasts from systems of interdependent equations, obtained from reduced-form equations, ought to be determined from the variance matrices and the covariances of the structural parameter estimations from structural-form equations (Wiśniewski, 2017).

A prediction based on a model's reduced-form equations has, in a sense, optimal properties, if an appropriate estimation method was used to estimate the parameters (Pawłowski, 1973, p. 254; Wiśniewski and Zieliński, 2004, p. 374). A prediction based on reduced-form equations has the quality of optimality, in a sense, that it gives lesser average prediction errors than the other methods, using the same amount of information.

In this work, the procedure for constructing econometric prognoses will be focused on a model with a feedback between the average salary in a medium-sized commercial and service enterprise (APAY) and the labor efficiency (EFEMP), as presented below.

Fig. 1 Scheme of the APAY-EFEMP model



Source: Own elaboration.

Statistical data are derived from a medium-sized commercial and service enterprise. They were used previously to present the econometric model of the medium enterprise (Wiśniewski, 2016b, p. 53) and have the form of quarterly time series. The hypothetical system of interdependent equations is written by formulas (1)-(2):

	$EFEMP = \alpha_{10} + \beta_{12}APAY + \alpha_{12}t - \lambda_{11}Q_1 - \gamma_{12}Q_2 + \eta_{Es},$	(1)
	$APAY = \alpha_{20} + \beta_{21}EFEMP + \alpha_{21}APAY_{-1} + \eta_{As},$	(2)

where:
$APAY_{-1}$ – the average quarterly pay delayed by 1 trimester,

t – the time variable (t = 1, …, 32),
Q_1 – the dummy variable taking the value of 1 in each first quarter and zero in the remaining quarters,
Q_2 – the dummy variable taking the value of 1 in each first quarter and zero in the remaining quarters,
α_{10}, α_{20} – the constant terms of the equations,
α_{12}, α_{21}, β_{12}, β_{21}, γ_{11}, γ_{12} – the structural parameters of the equations,
η_{Es}, η_{As} – the random components of the structural-form equations.

The reduced-form equations are given by formulas (3) and (4):

$$EFEMP = \pi_{10} + \pi_{11}APAY_{-1} + \pi_{12}t - \pi_{13}Q_1 - \pi_{14}Q_2 + \eta_{Er}, \tag{3}$$

$$APAY = \pi_{20} + \pi_{21}APAY_{-1} + \pi_{22}t - \pi_{23}Q_1 - \pi_{24}Q_2 + \eta_{Ar}, \tag{4}$$

where:
Π_{1j}, Π_{2j} – the structural parameters of the reduced-form equations (j = 0, 1, …, 4),
η_{Er}, η_{Ar} – the random components of the reduced-form equations.

In the literature on econometric forecasting, numerous works on specific forecasting areas can be found (e.g. Armstrong and Brodie, 1999; Armstrong, 2006; Hetemäki and Mikkola, 2017; Litterman, 1986; Pinheiro, 2013; Ravishankar et al., 1991). Alternatively, works about the history or the philosophy of forecasting have emerged (e.g. Clemen, 1989; Hawkins, 2015; Kim et al., 2006). There are no works, however, in the area of multi-equation econometric micromodels used for econometric forecasting.

Tab. 1 The average pay and performance in ENERGY enterprise quarterly in the years 2008-2013

Quarter	*APAY*	*EFEMP*	$APAY_{-1}$	*t*	Q_1	Q_2	Q_3
08_1	11.33	52.83	-	1	1	0	0
08_2	11.31	79.16	11.33	2	0	1	0
08_3	10.94	83.07	11.31	3	0	0	1
08_4	12.25	90.80	10.94	4	0	0	0
09_1	11.33	68.59	12.25	5	1	0	0
09_2	12.23	97.80	11.33	6	0	1	0
09_3	10.70	94.76	12.23	7	0	0	1
09_4	11.08	102.41	10.70	8	0	0	0
10_1	9.39	52.28	11.08	9	1	0	0
10_2	9.85	86.96	9.39	10	0	1	0
10_3	11.10	103.89	9.85	11	0	0	1
10_4	14.54	119.29	11.10	12	0	0	0
11_1	15.34	70.16	14.54	13	1	0	0
11_2	12.77	89.66	15.34	14	0	1	0
11_3	13.93	149.73	12.77	15	0	0	1
11_4	15.75	154.69	13.93	16	0	0	0
12_1	10.19	96.55	15.75	17	1	0	0
12_2	9.98	129.76	10.19	18	0	1	0
12_3	12.14	137.75	9.98	19	0	0	1
12_4	12.83	156.43	12.14	20	0	0	0
13_1	19.89	91.20	12.83	21	1	0	0
13_2	24.01	155.25	19.89	22	0	1	0
13_3	24.98	187.40	24.01	23	0	0	1
13_4	24.37	199.54	24.98	24	0	0	0

Source: Own calculations based on the data from the work Wiśniewski (2016a, p. 53)

3. Results and Discussion

The parameters of equations (1), (2), (3) and (4) were estimated using the least squares method, applying the data from Wiśniewski (2016b, p. 53) and table 1. The validity for using this estimation method results from its significantly higher precision (efficiency), in comparison to the double least squares method (Wiśniewski, 2011).

Empirical equations of the structural form (5) and (6) are as follows:

$$\underset{}{\widehat{EFEMP}} = \underset{(6.039)}{59.97} + \underset{(2.529)}{2.131}APAY + \underset{(5.383)}{3.01}t - \underset{(6.934)}{48.68}Q_1 - \underset{(2.604)}{18.15}Q_2 + u_{Es}, \tag{5}$$

$$R^2_{Es} = 0.901,\ Su_{Es} = 13.86,\ DW_{Es} = 1.464,$$

$$\widehat{APAY} = -0.0015 + 0.0285EFEMP + 0.802APAY_{-1} + u_{As}, \quad (6)$$
$$(0.001) \quad (1.661) \quad (5.068)$$

$$R^2_{As} = 0.770, Su_{As} = 2.399, hD_{As} = 1.727.$$

The following markings occur in the above equations:
u_{Es}, u_{As} – the residuals of corresponding structural-form equations,
R_{Es}^2, R_{As}^2 – the values of the determination coefficients of the equations,
Su_{Es}, Su_{As} – the standard errors of equation residuals,
DW_{Es}, hD_{As} – the values of the Durbin-Watson statistic and the Durbin h statistic, respectively.

The empirical reduced-form equations have the following form:

$$\widehat{EFEMP} = 53.841 + 2.490APAY_{-1} + 3.244t - 53.348Q_1 - 18.48Q_2 + u_{Er}, \quad (7)$$
$$(5.727) \quad (3.066) \quad (6.403) \quad (7.984) \quad (2.932)$$

$$R^2_{Er} = 0.915, Su_{Er} = 12.54, DW_{Er} = 1.873,$$

$$\widehat{APAY} = 1.366 + 0.794APAY_{-1} + 0.173t - 0.945Q_1 - 0.337Q_2 + u_{Ar}, \quad (8)$$
$$(0.742) \quad (4.980) \quad (1.744) \quad (0.720) \quad (0.273)$$

$$R^2_{Ar} = 0.782, Su_{Ar} = 2.462, DW_{Ar} = 1.549.$$

Equations (7) and (8) have markings analogous to the system (5), (6), which denote, respectively, the residuals, the coefficients of determination, the standard errors of residuals and the values of the Durbin-Watson statistic.

Forecasting begins with the reduced-form equation. Based on equations (7) and (8), forecasts of the labor efficiency and the average pay in the enterprise were determined for the quarter numbered T = 29. The forecasts of these forecasted variables are as follows: $EFEMP_{Tp}^{(r)}$ = 142.265 thousand PLN/1 employee, while $APAY_{Tp}^{(r)}$ = 24.101 thousand PLN/1 employee.

Forecasting from the structural-form equations begins with the use of an already exiting forecast $EFEMP_{Tp}^{(r)}$ = 142.265. Using equation (6), the first forecast from the structural-form is obtained, in which the value of the inserted labor efficiency is $EFEMP_{Tp}^{(r)}$ = 142.265. As such, we get a forecast of the average pay $APAY_{Tp}^{(s)}$ = 23.601 thousand PLN/1 employee. The obtained forecasts of the average monthly pay in the enterprise clearly are different ($APAY_{Tp}^{(s)} = 23.601 \neq APAY_{Tp}^{(r)} = 24.101$). This signifies a lack of synchronization of the $APAY_{Tp}^{(s)}$ = 23.601 forecast with the labor efficiency forecast $EFEMP_{Tp}^{(r)}$ = 142.265. It is thus necessary to determine the labor efficiency forecast from equation (5), using the average pay forecast from the structural-form equation $APAY_{Tp}^{(s)}$ = 23.601. The forecast obtained is: $EFEMP_{Tp}^{(s)}$ = 136.890 thousand PLN/1 employee. Differences between the prognoses: $EFEMP_{Tp}^{(s)} = 136.890 \neq EFEMP_{Tp}^{(r)} = 142.265$ can be noticed. As such, it is necessary to calculate the average pay forecast, with an assumption that labor efficiency will reach the value of $EFEMP_{Tp}^{(s1)}$ = 136.890 thousand PLN/1 employee. Using equation (6), a forecast of the average pay: $APAY_{Tp}^{(s2)}$ = 23.448 is obtained. The forecast of the average monthly pay is still different than the value obtained from the previous iteration ($APAY_{Tp}^{(s1)} = 23.601 \neq APAY_{Tp}^{(s2)} = 23.448$).

The procedure of correcting the forecasts should be continued up to the moment of obtaining the first repetition of the forecast result in the iteration P (p = 0, 1, …, P). So, if a situation occurs when $APAY_{Tp}^{(sp)} = APAY_{Tp}^{(sp\text{-}1)}$, the forecasting procedure should be finished. At that point, stabilization of the forecast of the variable remaining in feedback takes place. In our case, a situation occurs, when the labor efficiency forecasts fulfill the condition: $EFEMP_{Tp}^{(sp)} = EFEMP_{Tp}^{(sp+1)}$. Thus, convergence of the forecasts is obtained, which signifies a full feedback between EFEMP and APAY.

In the case in question, convergence of the feedbacks was obtained after four iterations ($P = 4$) of the calculations from the structural form of the econometric model (p = 0, 1, …, 4). The forecasts of the variables EFEMP and APAY in subsequent iterations are presented in table 2. Worthiness of conducting a third and a fourth iteration ought to be considered though. In practice, the differences between the forecasts obtained in third and fourth iterations – from a practical point of view – are not relevant in the process of enterprise management. However, without calculations in the third and the fourth iteration, the final forecasting result will not be known.

Tab. 2 Forecasts of the variables APAY and EFEMP, obtained from the structural-form, in subsequent iterations of the procedure

Predictor equation	**Iteration from the structural form**				
	0	1	2	3	4
APAY	23.601	23.448	23.439	23.438	23.438
EFEMP	142.265	136.890	136.564	136.545	136.543

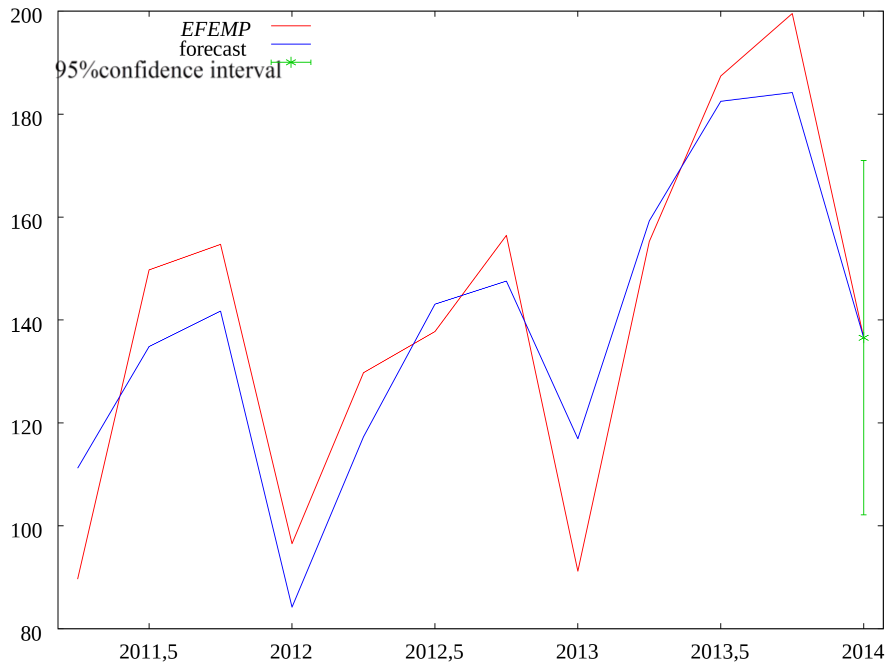
Source: Own calculations based on the information from the company

Tab. 3 Comparison of the structural-form forecasts with the reduced-form forecasts

Predictor equation	Reduced-form forecasts	Structural-form forecasts	Difference
APAY	24.101	23.438	-0.663 (-2.83%)
EFEMP	142.265	136.543	-5.722 (-4.19%)

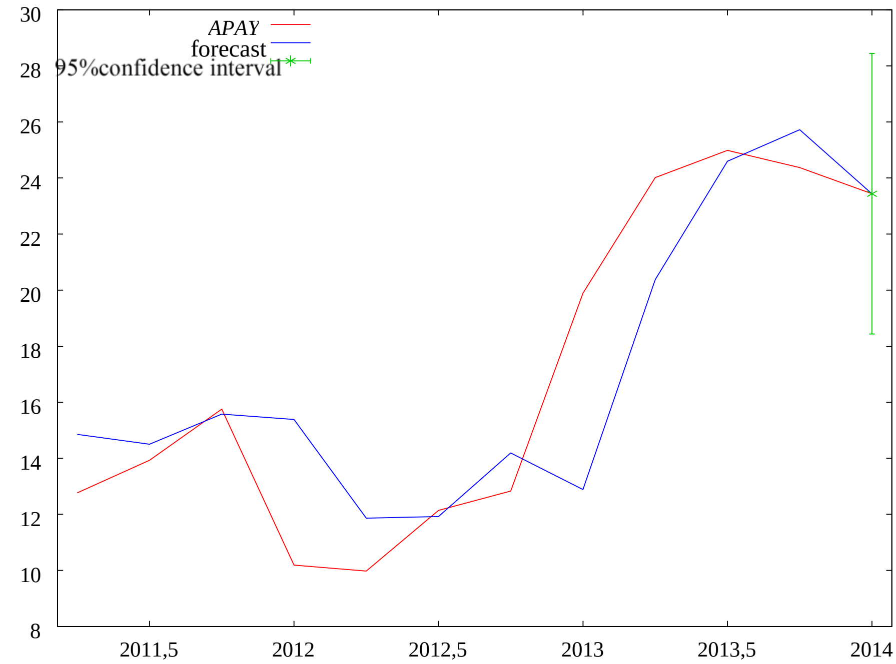
Source: Own calculations based on the information from the company

Fig. 1 The labor efficiency forecast $EFEMP_{Tp}^{(s4)}$



Source: Own calculations based on the company's information

Fig. 2 The average pay forecast $APAY_{Tp}^{(4s)}$



Source: own calculations based on the company's information

Table 3 presents the forecasts of the average pay and the labor efficiency obtained from the reduced-form and the structural-form equations. Comparison of these forecasts reveals significant differences between the results from the reduced form and those from the structural-form. The forecasts from the reduced form clearly are overestimated. The forecast of the average monthly pay, obtained from the reduced form, is overestimated nearly by 3%, in comparison with the forecast from the structural-form equation. In turn, the labor efficiency forecast from the reduced-form equation is overestimated, in relation to the forecast from the structural-form, by over 4%. The forecasts of the average pay and the labor efficiency are shown on figures 1 and 2.

4. Conclusions

The forecasting proceeding presented in this work effectively leads to obtaining convergent forecasts from the structural form of a system of interdependent equations. The final results obtained in such way, signify synchronization of the forecasts of the variables remaining in feedback. Forecasts from the reduced-form, at the values of the coefficients of determination $R^2 < 0.9$, are characterized by a lack of synchronization within the feedback.

It seems necessary to further investigate the efficiency of the proposed method of econometric forecasting from structural-form equations. Without empirical studies, it is not certain that it is possible to reach a pair of convergent predictions in each case.

It is also necessary to find the answer to the question whether the final result of forecasting depends on the selection of the reduced-form equation, which the “breaking” of the feedback begins with. Based on the current scant practice, this selection does not influence the final values of the forecasts. Basically, however, the procedure begins with the reduced-form equation that has the highest value of R^2.

The problem of multi-equation models falls within the theory of econometrics and applied econometrics. The empirical forecasting based on multi-equation models is not often presented in literature. Particularly, there are very little works on forecasting from systems of interdependent equations, especially econometric micromodels describing a specific business entity.

Little interest in the problems of econometric micromodels describing an enterprise has been observed on the part of researchers. The main reason is the lack of access to statistical data at an enterprise level. Empirical works in the area of financial econometrics dominate. The only data available is the statistical data from stock exchanges or financial markets, which satisfies the “hunger” for econometric research. However, empirical econometrics ought to be enriched with studies on specific enterprises.

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