ORIENTED BEHAVIOURAL PRESENT VALUE – REVISED APPROACH

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Abstract: *In general, behavioural present value (BPV) is such kind of present value which is depended of behavioural factors. The starting point for our discussion is BPV defined as a L-R fuzzy number. The main goal of this paper is to describe oriented BPV by means of revised notion of ordered fuzzy number (OFN). Then the information described by BPV is supplemented with a subjective forecast of the orientation of the market price trend. This forecast is implemented in the model of oriented BPV as an orientation of OFN. The presumption of market price increase is described by the positive orientation of OFN. The presumption of market price decrease is described by the negative orientation of OFN. In subject literature it is shown that, the fuzzy discount factor is such portfolio analysis tool which is better than return rate. Therefore, at the end, the expected fuzzy discount factor is determined under the assumption that future value is a random variable under Gaussian distribution of probability and present value is given as oriented BPV. It is shown that the orientation of expected fuzzy discount factor is consistent with orientation of the oriented BPV defining it.*

Key words: *behavioural present value, ordered fuzzy number, discount factor*

JEL codes: *C44, C02, G10*

1. Introduction

This work belongs to the mainstream of research presented, inter alia, in (Łyczkowska-Hanćkowiak, 2017; Łyczkowska-Hanćkowiak, Piasecki, 2018a, 2018b; Piasecki, 2011, 2014, 2015, 2016, 2017; Piasecki, Siwek, 2015, 2018; Siwek, 2017).

The present value (PV) is called the current equivalent value of payments at a fixed point time (Piasecki, 2011). The present value of the future cash flow may be imprecise. For this reason, the PV is described using fuzzy numbers.

In Piasecki (2011) and Piasecki and Siwek (2015) the behavioural present value (BPV) was defined as such approximation of current market price which is imprecisely estimated under impact of behavioural factors. In (Łyczkowska-Hanćkowiak, 2017) the information described by BPV is supplemented with a subjective forecast of the market price trend. This forecast was implemented in the model of oriented BPV as an orientation of fuzzy number. In this way the BPV was replaced by oriented BPV defined by an ordered fuzzy number (Kosiński et al., 2003). In this paper oriented BPV is described with use the revised definition of oriented fuzzy number (Piasecki, 2018). The positive orientation of fuzzy number describes a subjective prediction of rise in market price. The negative orientation of fuzzy number describes a subjective prediction of fall in market price. An example illustrates the new form of oriented BPV. In Piasecki and Siwek (2018) it is shown that, for appraising the considered securities, the expected fuzzy discount factor is very good tool for portfolio analysis. Therefore, at the end, oriented fuzzy discount factor is determined by means of oriented BPV.

2. Elements of ordered fuzzy number theory

An imprecise number is a family of values in which each considered value belongs to it in a varying degree. A commonly accepted model of imprecise number is the fuzzy number (FN), defined as a fuzzy subset of the real line $\mathbb{R}$. The most general definition of FN is given by Dubois and Prade (1978). Dubois and Prade (1980) distinguished a special type of representation of FN called LR-type FN defined as follow. For any non-decreasing sequence $\{a, b, c, d\} \subset \mathbb{R}$ the left reference function $L_S: [a, b] \to [0,1]$ and the right reference function $R_s: [c, d] \to [0,1]$ are upper semi-continuous monotonic functions satisfying the condition $L_S(b) = R_S(c) = 1$. Then the membership function $\mu_S(\cdot\,|a, b, c, d, L_S, R_S) \in [0,1]^{\mathbb{R}}$ of the FN $\mathcal{S}(a, b, c, d, L_S, R_S)$ (which is called LR-type FN) is defined by the identity

$$\mu_S(x|a, b, c, d) = \begin{cases} 0, & x \notin [a, d] = [d, a], \\ L_S(x), & x \in [a, b] = [b, a], \\ 1, & x \in [b, c] = [c, b], \\ R_S(x), & x \in [c, d] = [d, c].^{1} \end{cases} \tag{1}$$

The concept of ordered fuzzy numbers (OFN) was intuitively introduced by Kosiński et al. (2002) as an extension of the concept of FN. A significant drawback of Kosiński's theory is that there exist such OFNs which, in fact, are not FN. What is more, the intuitive Kosiński's approach to the notion of OFN is very useful. For this reason, the Kosiński's theory was revised in Piasecki (2018). The OFNs' usefulness follows from the fact that an OFN is a FN supplemented by orientation which is understood as a linear order on the real line $\mathbb{R}$. Mentioned orientation may be negative or positive. Negative orientation means a linear order on $\mathbb{R}$ from bigger numbers to smaller ones. Negative oriented number is interpreted as such number, which may decrease. Positive orientation means a linear order on $\mathbb{R}$ from smaller numbers to bigger ones. Positive oriented number is interpreted as such number, which may increase. We will denote any orientation from $a \in \mathbb{R}$ to $b \in \mathbb{R}$ by the symbol $[\![a \rightarrowtail b]\!]$.

For any monotonic sequence $\{a, b, c, d\} \subset \mathbb{R}$ the starting-function $L_S: [a, b] \longrightarrow [0,1]$ and the ending-function $R_S: [c, d] \longrightarrow [0,1]$ are continuous from above monotonic functions satisfying the condition $L_S(b) = R_S(c) = 1$.

The identity (1) defines the membership function $\mu_{\overleftrightarrow{S}}(\cdot\,|a, b, c, d, L_S, R_S) \in [0,1]^{\mathbb{R}}$ of the OFN $\overleftrightarrow{S}(a, b, c, d, L_S, R_S)$. The space of all OFN we denote by the symbol $\mathbb{K}$.

The condition $a < d$ fulfilment determines the positive orientation $[\![a \rightarrowtail d]\!]$ of OFN $\overleftrightarrow{S}(a, b, c, d, L_S, R_S)$. In this case, the starting-function L_S is non-decreasing and the ending-function R_S is non-increasing. Any positively oriented OFN is interpreted as such imprecise number, which may increase. The condition $a > d$ fulfilment determines the negative orientation $[\![a \rightarrowtail d]\!]$ of OFN $\overleftrightarrow{S}(a, b, c, d, L_S, R_S)$. In this case, the starting-function L_S is non-increasing and the ending-function R_S is non-decreasing. Negatively oriented OFN is interpreted as such imprecise number, which may decrease. For the case $a = d$, OFN $\overleftrightarrow{S}(a, a, a, a)$ represents crisp number $a \in \mathbb{R}$, which is not oriented.

3. Oriented fuzzy present value

The present value (PV) is defined as a present equivalent of a cash flow in a given time in the present or future. It is commonly accepted that the PV of a future cash flow can be imprecise. The natural consequence of this approach is estimating PV with fuzzy numbers. A detailed description of the evolution of this particular model can be found in (Piasecki, 2014). Among other things, imprecise PV may be evaluated by OFN (Łyczkowska-Hanćkowiak, Piasecki, 2018a). Then PV is characterized by monotonic sequence $\{V_s, V_f, \check{C}, V_l, V_e\}$, where:

- $\check{C}$ – market price,
- $[V_s, V_e] \subset \mathbb{R}^+$ is interval of all possible PV' values,
- $[V_f, V_l] \subset [V_s, V_e]$ is interval of all prices which do not perceptible differ from market price $\check{C}$.

The oriented PV (OPV) is estimated by OFN:

$$\overleftrightarrow{PV} = \overleftrightarrow{S}(V_s, V_f, V_l, V_e, L_{PV}, R_{PV}), \tag{2}$$

where the left reference function $L_{PV}: [V_s; V_f] \to [0; 1]$ and the right reference function $R_{PV}: [V_l; V_e] \to [0; 1]$ are given ones. If we predict a rise in market price then OPV is described by positively oriented OFN. If we predict a fall in market price, then OPV is described by negatively oriented OFN.

4. Oriented fuzzy behavioural present value

Let us consider any financial instrument in which is the subject of trade on the highly effective financial market. The market price of this instrument may fluctuate over time. Therefore, we can talk about a market price trend. Financial equilibrium is the state of the financial market in which the trend of market price is constant. The value of market price $\check{C}$ is equal to the equilibrium price C_0 determined by a technical or fundamental analysis. Piasecki (2011) defined PV as the utility function of financial flow. This PV depends on both subjective and objective conditions. Hence PV deviation from the market price is imprecise. Behavioural present value (BPV)

[1] Let us note that this identity describes additionally extended notation of numerical intervals, which is used in this work.

depends on selected behavioural factors. In Piasecki (2011) and Piasecki and Siwek (2015), BPV was determined as LR-type FN given by following membership function μ_{BPV}:

- for $\Delta C > 0$:

$$\mu_{BPV}(x|\Delta C > 0) = \begin{cases} \frac{(x-V_{min})(1+\delta C)}{\check{C}-V_{min}+(x-V_{min})\delta C} & \text{for } x \in [V_{min}; \check{C}] \neq \{\check{C}\}, \\ \frac{V_{max}-x}{V_{max}-\check{C}+(V_{max}-x)\delta C} & \text{for } x \in]\check{C}; V_{max}], \\ 0 & \text{for } x \notin [V_{min}; V_{max}], \end{cases} \tag{3}$$

- for $\Delta C \leq 0$:

$$\mu_{BPV}(x|\Delta C \leq 0) = \begin{cases} \frac{x-V_{min}}{\check{C}-V_{min}+(x-V_{min})\delta C} & \text{for } x \in [V_{min}; \check{C}[, \\ \frac{(V_{max}-x)(1+\delta C)}{V_{max}-\check{C}+(V_{max}-x)\delta C} & \text{for } x \in [\check{C}; V_{max}] \neq \{\check{C}\}, \\ 0 & \text{for } x \notin [V_{min}; V_{max}], \end{cases} \tag{4}$$

where:

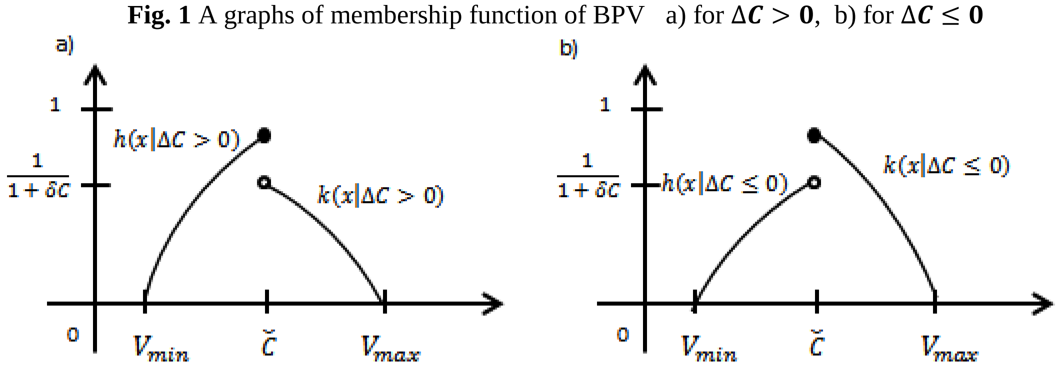
- V_{min} – the maximal lower evaluation of the PV,
- V_{max} – the minimal upper evaluation of the PV,
- $\check{C}$ – the observed market price,
- $\delta C = \frac{|\Delta C|}{\check{C}} = \frac{|\check{C}-C_0|}{\check{C}}$ - relative deviation of the market price from the equilibrium price,
- C_0 – the substantively justified equilibrium price.

In general, BPV is such a fuzzy number, which is an approximation of the market price $\check{C}$. BPV can be described as LR- type FN

$$BPV = \mathcal{S}(V_{min}, \check{C}, \check{C}, V_{max}, h, k) \tag{5}$$

determined by its membership function μ_{BPV} determined separately for the cases $\Delta C > 0$ and $\Delta C \leq 0$. Figure 1 shows a graphs of the membership function $\mu_{BPV}(\cdot|\Delta C > 0)$ and $\mu_{BPV}(\cdot|\Delta C \leq 0)$ depending on the deviation of the market price from the equilibrium price.

Fig. 1 A graphs of membership function of BPV a) for $\Delta\boldsymbol{C} > \mathbf{0}$, b) for $\Delta\boldsymbol{C} \leq \mathbf{0}$



Source: Own elaboration.

If we take into account subjective predictions of future price changes, we describe oriented BPV (OBPV) by OFN:

$$\overleftrightarrow{BPV} = \overleftrightarrow{\mathcal{S}}(V_s, \check{C}, \check{C}, V_e, L_{BPV}, R_{BPV}) \tag{6}$$

where:

- $[V_s, V_e] = [V_{min}, V_{max}] \subset \mathbb{R}^+$ is interval of all possible BPV' values,
- (L_{BPV}, R_{BPV}) is any ordered pair of starting-function and ending-function.

The OFBPV $\overleftrightarrow{BPV}$ is determined by its membership function $\mu_{\overleftrightarrow{BPV}}$ given by the identity (3) or (4). The presumption of market price increase is described by the positive orientation of OBPV. The presumption of market price decrease is described by the negative orientation of OBPV.

We obtain four cases of OBPV. If the market price surpasses the equilibrium price i.e. $\Delta C > 0$ and we subjectively predict rise in market price then OBPV is described by positively oriented OFN

$$\overleftrightarrow{BPV} = \overleftrightarrow{\mathcal{S}}\left(V_{min}, \check{C}, \check{C}, V_{max}, h(\cdot|\Delta C > 0), k(\cdot|\Delta C > 0)\right). \tag{7}$$

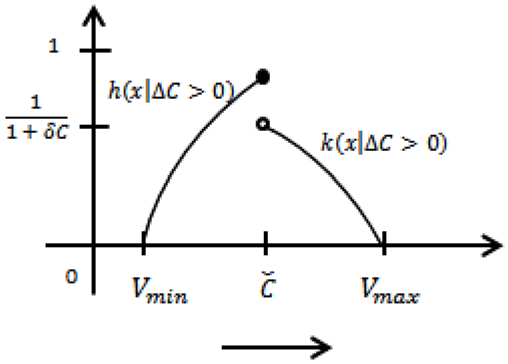
For this case, the graph of membership function is presented in figure 2.

If the market price surpasses the equilibrium price i.e. $\Delta C > 0$ and we subjectively predict fall in market price then OBPV is described by negatively oriented OFN:

$$\overleftrightarrow{BPV} = \overleftrightarrow{S}\left(V_{max}, \check{C}, \check{C}, V_{min}, k(\cdot\,|\Delta C > 0\,), h(\cdot\,|\Delta C > 0\,)\right). \tag{8}$$

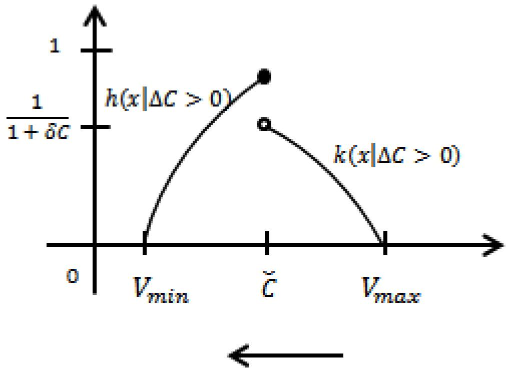
For this case, the graph of membership function is presented in figure 3.

Fig. 2 A graph of membership function of positively oriented BPV if $\boldsymbol{\Delta C > 0}$



Source: Own elaboration.

Fig. 3 A graph of membership function of negatively oriented BPV if $\boldsymbol{\Delta C > 0}$



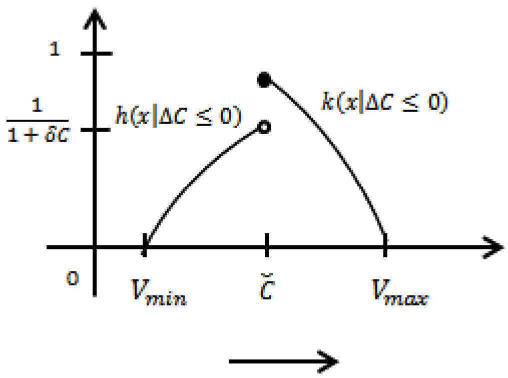
Source: Own elaboration.

If the equilibrium price surpasses the market price i.e. $\Delta C \leq 0$ and we subjectively predict rise in market price then OBPV is described by positively oriented OFN:

$$\overleftrightarrow{BPV} = \overleftrightarrow{S}\left(V_{min}, \check{C}, \check{C}, V_{max}, h(\cdot\,|\Delta C \leq 0\,), k(\cdot\,|\Delta C \leq 0\,)\right). \tag{9}$$

For this case, the graph of membership function is presented in figure 4.

Fig. 4 A graph of membership function of positively oriented BPV for the case $\boldsymbol{\Delta C \leq 0}$



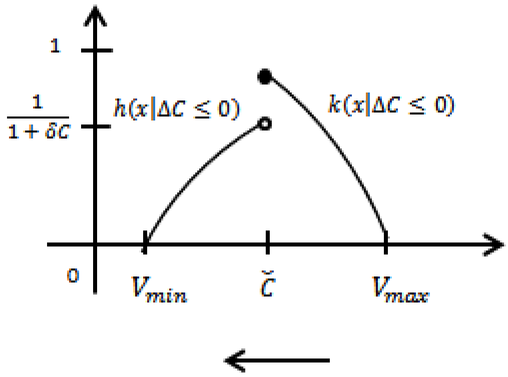
Source: Own elaboration

If the equilibrium price surpasses the market price i.e. $\Delta C \leq 0$ and we subjectively predict fall in market price then OBPV is described by negatively oriented OFN:

$$\overleftrightarrow{BPV} = \overleftarrow{\mathcal{S}}\left(V_{max}, \check{C}, \check{C}, V_{min}, k(\cdot\,|\Delta C \leq 0\,), h(\cdot\,|\Delta C \leq 0\,)\right). \tag{10}$$

For this case, the graph of membership function is presented in figure 5.

Fig. 5 A graph of membership function of negatively oriented BPV for the case $\boldsymbol{\Delta C \leq 0}$



Source: Own elaboration.

Example 1. For fixed security we observe market price $\check{C} = 60$. Substantially justified the equilibrium price is given as follows $C_0 = 40$, the observed market price $\check{C} = 60$. The maximal lower and the minimal upper evaluation of PV are respectively $V_{min} = 30$ and $V_{max} = 80$. We have:

$$\Delta C = \check{C} - C_0 = 60 - 40 = 20 > 0\,,$$

$$\delta C = \frac{|\Delta C|}{\check{C}} = \frac{20}{60} = \frac{1}{3}.$$

Additionally, we predict fall in market price. Then OBPV is described as negatively oriented OFN:

$$\overleftrightarrow{BPV} = \mathcal{S}\left(80; 60; 60; 30; k(\cdot\,|20\,); h(\cdot\,|20\,)\right),$$

Where:

$$h(x) = h(x|20) = \frac{4x-120}{x+60} \qquad \text{for} \quad x \in [60; 30],$$

$$k(x) = k(x|20) = \begin{cases} \frac{3x-240}{x-140} & \text{for} \quad x \in [80; 60[, \\ 1 & \text{for} \qquad x = 60\,. \end{cases}$$

It means that considered OBPV is explicitly determined by its membership function:

$$\mu_{BPV}(x|20) = \begin{cases} \frac{3x-240}{x-140} & \text{for} \quad x \in [80; 60], \\ \frac{4x-120}{x+60} & \text{for} \quad x \in [60; 30]\,, \\ 0 & \text{for} \quad x \notin [80; 30]. \end{cases} \qquad \square$$

5. Oriented fuzzy discount factor

Let us assume that the time horizon $t > 0$ of an investment is fixed. Then, the security considered here is determined by two values: anticipated FV V_t and assessed PV V_0. The basic characteristic of benefits from owning this security is the simple return rate defined as:

$$r_t = \frac{V_t - V_0}{V_0} = \frac{V_t}{V_0} - 1. \tag{11}$$

In practice of financial markets analysis, the uncertainty risk is usually described by probability distribution of return rate calculated for $V_0 = \check{C}$. After Markowitz (1952) we assume that this simple return rate has the Gaussian distribution $N(\bar{r}, \sigma)$. Then the expected discount factor $\bar{v} \in \mathbb{R}$ is given by the identity:

$$\bar{v} = \frac{1}{1+\bar{r}}. \tag{12}$$

In Łyczkowska-Hanćkowiak and Piasecki (2018b) the expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is considered for the case $V_0 = \overleftrightarrow{PV}$, where PV is determined by (2). The results obtained there imply that if we have:

$$V_0 = \overleftrightarrow{BPV} = \overleftrightarrow{\mathcal{S}}(V_s, \check{C}, \check{C}, V_e, L_{BPV}, R_{BPV}) \tag{13}$$

then the expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described by OFN:

$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}\left(\frac{V_s \cdot \bar{v}}{\check{C}}, \bar{v}, \bar{v}, \frac{V_e \cdot \bar{v}}{\check{C}}, L_V, R_V\right), \tag{14}$$

Where:

$$L_V(x) = L_{BPV}\left(\frac{x \cdot \check{C}}{\bar{v}}\right), \tag{15}$$

$$R_V(x) = R_{BPV}\left(\frac{x \cdot \check{C}}{\bar{v}}\right). \tag{16}$$

Let us substitute now

$$\eta(x) = h\left(\frac{x \cdot \check{C}}{\bar{v}}\right), \tag{17}$$

$$\kappa(x) = k\left(\frac{x \cdot \check{C}}{\bar{v}}\right). \tag{18}$$

Due (14) we obtain following conclusions:

- if $\overleftrightarrow{BPV}$ is given by (7) then expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described as the OFN:
$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}\left(\frac{V_{min} \cdot \bar{v}}{\check{C}}, \bar{v}, \bar{v}, \frac{V_{max} \cdot \bar{v}}{\check{C}}, \eta(\cdot\,|\Delta C > 0), \kappa(\cdot\,|\Delta C > 0)\right), \tag{19}$$
- if $\overleftrightarrow{BPV}$ is given by (8) then expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described as the OFN:
$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}\left(\frac{V_{max} \cdot \bar{v}}{\check{C}}, \bar{v}, \bar{v}, \frac{V_{min} \cdot \bar{v}}{\check{C}}, \kappa(\cdot\,|\Delta C > 0), \eta(\cdot\,|\Delta C > 0)\right), \tag{20}$$
- if $\overleftrightarrow{BPV}$ is given by (9) then expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described as the OFN:
$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}\left(\frac{V_{min} \cdot \bar{v}}{\check{C}}, \bar{v}, \bar{v}, \frac{V_{max} \cdot \bar{v}}{\check{C}}, \eta(\cdot\,|\Delta C \le 0), \kappa(\cdot\,|\Delta C \le 0)\right), \tag{21}$$
- if $\overleftrightarrow{BPV}$ is given by (10) then expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described as the OFN:
$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}\left(\frac{V_{max} \cdot \bar{v}}{\check{C}}, \bar{v}, \bar{v}, \frac{V_{min} \cdot \bar{v}}{\check{C}}, \kappa(x \cdot |\Delta C \le 0), \eta(\cdot\,|\Delta C \le 0)\right). \tag{22}$$

Let us note that the orientation of expected discount factor is always identical with orientation of the BPV determining it.

Example 2. For the security described in the Example 1 its simple return rate has normal distribution $N(0.25{,}0.1)$. Then expected discount factor $\overleftrightarrow{\mathcal{V}} \in \mathbb{K}$ is described as negatively oriented OFN:

$$\overleftrightarrow{\mathcal{V}} = \overleftrightarrow{\mathcal{S}}(1.0667; 0.8; 0.8;\ 0.4;\ \kappa(\cdot\,|20);\ \eta(\cdot\,|20)),$$

where

$$\eta(x) = \eta(x|20) = \frac{300 \cdot x - 120}{75 \cdot x + 60} \qquad \text{for} \quad x \in [0.8; 0.4],$$

$$\kappa(x) = \kappa(x|20) = \begin{cases} \frac{225 \cdot x - 240}{75 \cdot x - 140} & \text{for} \quad x \in [1.0667; 0.8[, \\ 1 & \text{for} \quad x = 0.8. \end{cases}$$

It means that considered OBPV is explicitly determined by its membership function:

$$\mu_{BPV}(x|20) = \begin{cases} \frac{225 \cdot x - 240}{75 \cdot x - 140} & \text{for} \quad x \in [1.0667; 0.8], \\ \frac{300 \cdot x - 120}{75 \cdot x + 60} & \text{for} \quad x \in [0.8; 0.4]\,, \\ 0 & \text{for} \quad x \notin [1.0667; 0.4]. \end{cases} \qquad \square$$

6. Conclusions

Summarizing, the behavioural present value was presented with an ordered fuzzy number. Fuzzy number which describes BPV has positive orientation when we expect (on the basis of subjective premises) the market price to increase. It has negative orientation contrary. The behavioural present value model can be a good tool for economic analysis and modeling. This allows you to assume that the results you get will be used for a portfolio analysis

The results of the work fully convince that the use of OFN will facilitate the analysis of financial instruments with imprecise estimated values. For example, obtained results may be applied in decision models described in Piasecki (2014). It is expedient to further development the fuzzy finance theory based on OFN.

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