THE CURRENCY TRADING SYSTEM WITH CONSTANT MAGNITUDE OF UNITARY RETURN

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Abstract: *In the article, we consider currency transactions related to the exchange rate given as the quotient of the base currency by the quoted currency. The unitary return is defined as the quotient of return expressed in quoted currency by the quote of base currency. All possible states of the currency market form a finite elemental space of states. All knowledge about the dynamics of the currency market is presented in the form of a prediction table describing the conditional probability distributions of incoming exchange rate changes with a given magnitude of the unitary return. On the other hand, in the proposed trading system each transaction is concluded in such a way that the gross payment is determined by the given magnitude of unitary return. Using the prediction table, we determine the success probabilities for each state of currency market. The set of decision-making premises is defined as the set of all those currency market states, for which the probability of success is greater than the value determined by the investor. The benefits of using the proposed transaction system are assessed using the expected annual payment. The investment risk is determined by means of expected Shannon's entropy. Both of these indexes can be used to select the optimal trading system.*

Key words: *prediction table, currency transactions' system, expected payment, Shannon's entropy*

JEL codes: *C61, F31, G11, G17*

1. Introduction

The idea of binary presentation of currency exchange rate was initially proposed in (de Villiers, 1933). This idea was used in following papers (Stasiak, 2016, 2017a, 2017b; Stasiak and Wójcicka, 2018) where various kinds of state models for binary representation were presented. Each of these models was associated with a trading system managing transactional decisions.

New, particular decision models resulted in the need to create a decision model which generalises all existing models using trading systems which base on prediction tables determined with the help of binary representation of currency exchange rates. The solution to this task will enable the future development of the theory and practise of binary modelling of currency markets. The main objective of the paper is to build such a model.

The creation of new decision models each time requires the assessment of utility and comparing this utility with the utility of existing models. Therefore, a part of the paper was dedicated to construct the criteria which are exclusively assigned to trading systems using prediction tables basing on binary representation of exchange rates.

2. A general model of currency market

The scope of the paper will be the market of a currency pair BCR/QCR noted in time $[0, T]$, where BCR means the base currency and QCR means the quoted currency. In each moment of time $t \in [0, T]$ we note the price Ask $Q_{Ask}(t)$ which is the price of base currency BCR in the quoted currency QCR. This way we determine the trend $Q_{Ask}: [0, T] \to \mathbb{R}$ which describes the dynamics of a currency pair BCR/QCR. The changes on the market are characterised by a unitary return defined as a quotient of a return expressed in QCR by the amount of the base currency. The unitary return $\mathcal{ur}(t', t'')$of a BUY transaction opened in time t' and closed in time $t'' > t'$ equals:

$$\mathcal{ur}(t', t'') = Q_{Ask}(t'') - Q_{Ask}(t'). \tag{1}$$

In the further presented trading system each operation opened in time $t' \in [0, T]$ will be closed as soon as possible in time $t'' > t'$ which fulfils the following condition:

$$|\mathcal{ur}(t', t'')| = \delta > 0, \tag{2}$$

where $\delta > 0$ is a given magnitude of unitary return. In such situation the transaction concluded in time $t' \in [0, T]$is closed in time $t'' > t'$ determined by the equation:

$$t'' = \min\{\min\{\tau > t': Q_{ask}(\tau) - Q_{ask}(t') = \delta\}, \min\{\tau > t': Q_{ask}(t') - Q_{ask}(\tau) = \delta\}\}. \tag{3}$$

Therefore, we can encounter only one out of the two following cases:

$$\min\{\tau > t': Q_{ask}(\tau) - Q_{ask}(t') = \delta\} < \min\{\tau > t': Q_{ask}(t') - Q_{ask}(\tau) = \delta\}, \tag{4}$$

$$\min\{\tau > t': Q_{ask}(\tau) - Q_{ask}(t') = \delta\} > \min\{\tau > t': Q_{ask}(t') - Q_{ask}(\tau) = \delta\}. \tag{5}$$

Closing the transaction in case (4) means the realisation of "take profit" strategy (TP) as there was the increase of exchange currency rate by δ. Such occurrence is denoted by $\mathcal{J}_\delta$. Closing the transaction in case (5) means the implementation of "stop loss" strategy (SL) as there was a decrease in currency rate. It is an opposite case to $\mathcal{J}_\delta$ and it is denoted as $\mathcal{J}_\delta^C$.

Describing the mechanism of currency market, we can distinguish the space $\mathbb{S} = \{s_j: j = 1,2, \dots, m\}$ of the possible states. Each opening moment of the transaction is attributed an observed state of currency market. These observations can be performed when using, for instance, the binary representation of currency rate defined for a discretisation unit equal a magnitude of unitary return δ (Stasiak, 2016). Those observations can also be conducted in another adequate method for a stated magnitude of unitary return δ. This way we can create a sequence $\{\tilde{s}_i\}_{i=1}^n$ of following observations of market states. Each state s_j is observed exactly $n_j > 0$ times. Thanks to this, for each state s_j we can establish the probability:

$$p(s_j) = \frac{n_j}{n} \tag{6}$$

of reaching that state by the currency rate pair of BCR/QCR. In the next step each state s_j is assigned a conditional probability:

$$p(\mathcal{J}_\delta | s_j) = \frac{n_j^*}{n_j} \tag{7}$$

of increase occurrence $\mathcal{J}_\delta$ due to reaching by currency market of BCR/QCR pair the state s_j. Also, here we can use the binary representation of currency exchange.

The set of triples $\{(n_j, p(s_j), p(\mathcal{J}_\delta | s_j)): j = 1,2, \dots, m\}$ forms the prediction table to foresee the changes of currency rate of BCR/QCR pair. Each state s_j can be a prediction premise. Total probability $p(\mathcal{J}_\delta)$ of currency rate increase can be denoted as:

$$p(\mathcal{J}_\delta) = \sum_{j=1}^m p(s_j) \cdot p(\mathcal{J}_\delta | s_j). \tag{8}$$

Example 1: In two years the changes of currency pair AUD/NZD were observed. The records were held for a given magnitude of unitary return $\delta = 20\ pips$ (it is $\delta = 0.00200$). The results are presented in table 1.□

The last row in table 1 shows as follows: a total number of observations, a probability of occurrence of any state and a total probability of the currency rate increase.

Tab. 1 Prediction table

State s_j	Number of observations n_j	Probability of the occurrence of state $p(s_j)$	Probability of the increase of a currency rate $p(\mathcal{I}_\delta \| s_j)$
s_1	1050	0.1613	0.5733
s_2	330	0.0507	0.5818
s_3	1285	0.1974	0.3953
s_4	653	0.1003	0.3492
s_5	1362	0.2092	0.5441
s_6	576	0.0885	0.6979
s_7	893	0.1372	0.3863
s_8	360	0.0553	0.4778
Total	6509	1.0000	0.4901

Source: Own calculations.

3. The description of Forex foreign exchange market

Forex exchange market is the place where the exchange rate of BCR/QCR is traded. The current value of the exchange rate is noted in two prices: Bid price (further denoted as Q_{Bid}) and Ask price (Q_{Ask}). Ask price is the price of the base currency BCR equivalent of QCR. Bid price is the selling price reflects how much of the quoted currency QCR will be obtained when selling one unit of the base currency BCR. Ask price is not lower than Bid price which can be showed as:

$$Q_{Ask} \geq Q_{Bid}. \tag{9}$$

Nowadays Ask and Bid price are noted with accuracy of 0.1 pip which is the accuracy of 0.00001.

The transaction on Forex foreign exchange market are entered by brokers of exchange market and investors. Forex brokers set their fees based on commission which equals the spread – the difference between a current Ask and Bid price. On Forex market of the BCR/QCR pair an investor can give the broker two orders: BUY and SELL. BUY order is executed with the Ask price Q_{Ask} and it means buying BCR with QCR. SELL order is executed with the Bid price Q_{Bid} and it means selling BCR with QCR. Each of the SEL and BUY orders can be an open or close order. The amount of the orders is defined by the investor and expressed in standard lot size. Except for the base currency of Japanese jen (JPY) one lot equals 100 000 units of base currency BCR.

An investor operating on Forex market of BCR/QCR pair wants to achieve a profit due to an accurate prediction of a change of a chosen currency pair exchange rate. To achieve that goal, an investor trades on Forex market. Each transaction consists of transaction opening in time $t'$and closing in a following time $t'' > t'$. The only opening or closing orders can be BUY or SELL orders. A broker accepting an opening order is obliged to undertake the closing order. In a moment t' of opening the transaction the levels of Ask and Bid prices are represented by a pair (Q'_{Ask}, Q'_{Bid}). Opening the order is accompanied by determining the value of the spread:

$$\overline{spr} = Q'_{Ask} - Q'_{Bid} \geq 0. \tag{10}$$

The value of a spread $\overline{spr}$ depends on the offer of the broker. In a moment $t''$of closing the transaction the levels of Ask and Bid prices are represented by a pair of (Q''_{Ask}, Q''_{Bid}). The transactions concluded on foreign exchange market are settled in QCR. Forex allows two positions: going long and going short.

If in the opening moment t' the investors expects the increase of the value of BCR/QCR pair then they give a BUY order in Ask price Q'_{Ask} of $\mathcal{v}$ value and they are going long. Going long means that in a determined time the investors places a SELL order with a Bid Q''_{Bid} price. Due to that transaction the investor gains:

$$y_l = \mathcal{v} \cdot (C''_{Bid} - C'_{Ask}). \tag{11}$$

If in opening moment t' the investors expect a decline in BCR/QCR pair, then they place a SELL order with a Bid price Q'_{Bid} of a $\mathcal{v}$ value and they go short. Taking a short position means that in a determined closing time t'' the investors places a closing BUY transaction with the Ask Q''_{Ask} price. Due to that transaction the investors obtain:

$$y_s = \mathcal{v} \cdot (C'_{Bid} - C''_{Ask}). \tag{12}$$

4. The proposed trading system

Since the market operates 24 hours 5 days a week placing the orders manually is tiresome and regarding short-term transactions just impossible. In such case, High-Frequency Trading systems are very helpful. HFT is a program trading platform that uses powerful computers to transact a large number of orders at very fast speeds which operate investment strategies.

In most HFT systems the signals to open or close a particular position are created via the analysis of ratios. A popular trading system places orders when two average lines intersect: short- and long-term average of quotations. The system bases on two decision-making rules:

- If the short-term average of quotations intersects with a higher long-term average, then place a BUY order;
- If the long-term average of quotations intersects with a higher short-term average, then place a SELL order;

Investment decisions made in above mentioned trading systems are characterised by a lack of explicitly determined potential payments. This lack makes the investment strategy analysis more difficult.

Regulated transactional systems, which have the prior stated payments, are devoid of that disadvantage. For each single transaction conducted by a regulated transactional system, one needs to state in advance as follows:

- The Q_{TP}price which closes the transaction with a profit (TP rule);
- The Q_{SL} price which closes the transaction with a loss (SL rule).

A proposal of a regulated transactional system which satisfies the (2) condition, will be presented below. According to that condition and the profitability criterion $\delta > \overline{\mathrm{spr}}$, the proposed system should also satisfy the following condition:

$$|Q''_{Ask} - Q'_{Ask}| = \delta > \overline{spr} \geq 0. \quad (13)$$

Then the decision-making premises is a prediction table presented in Section 2.

In a general case, the proposed transactional system is dependent on the given threshold $\overline{Thr} \geq 0.5$ and consists of three following decision rules used only if the state s_j occurs:

A) if the following condition is satisfied:

$$p(\mathcal{J}_\delta | s_j) > \overline{Thr}, \quad (14)$$

place a BUY order and go long.

B) if the following condition is satisfied:

$$1 - p(\mathcal{J}_\delta | s_j) > \overline{Thr}, \quad (15)$$

place a SELL order and go short.

C) if the transaction was opened with the Ask price C'_{Ask} and a current Ask price C''_{Ask} satisfies condition (13) – close the opened transaction.

The results of decision rules A) and B) are called recommendations.

A transactional system which depends on a multiple use of the above set of decision-making rules is called a discrete probabilistic transactional system denoted as DPT. DPT systems are a particular case of High Frequency Trading (HFT). (Li et al., 2015).

Each state s_j satisfying conditions (14) and (15) is called an acceptable decision-making premise. For a given decision threshold $\overline{Thr} > 0{,}5$ a set:

$$\mathbb{D}(\overline{Trh}) = \{s_j : \max\{p(\mathcal{J}_\delta | s_j), 1 - p(\mathcal{J}_\delta | s_j)\} > \overline{Trh}\} \subset \mathbb{S} \quad (16)$$

constitutes a set of all decision-making acceptable premises.

Let us consider potential payments possible to achieve in DPT system after opening the order with a value $\mathscr{v}$. Let's assume that there is an increase of the Ask price quotation. In such case in DPT system between the prices Ask and Bid the following dependencies occur:

$$Q'_{Bid} = Q'_{Ask} - \overline{spr}\ , \quad (17)$$

$$Q''_{Ask} = Q'_{ask} + \delta, \quad (18)$$

If the investor placed a SELL order, in this case according to (12) the payment will equal:

$$y_s = \mathscr{v} \cdot (-\delta - \overline{spr}) < 0. \quad (19)$$

In case of placing a BUY order the investor calculates its payment assuming a constant spread and predicts:

$$Q''_{bid} = Q'_{ask} + \delta - \overline{spr}. \quad (20)$$

According to these expectations and (11) the payment will be as follows:

$$y_l = \mathscr{v} \cdot (\delta - \overline{spr}) > 0. \quad (21)$$

Let us assume that there is a decline in the Ask price quotations. In this case in DPT system between the prices Ask and Bid the (17) and the following equation occur:

$$Q''_{Ask} = Q'_{ask} - \delta, \tag{22}$$

If the investor placed a SELL order, then in this case, according to (12) the payment will be:

$$y_s = \mathscr{v} \cdot (\delta - \overline{spr}) > 0. \tag{23}$$

In case of placing a BUY order, the investor calculates its profit assuming a constant spread and predicts:

$$Q''_{bid} = Q'_{ask} - \delta - \overline{spr}. \tag{24}$$

According to this prediction and (11) the profit will equal:

$$y_l = \mathscr{v} \cdot (-\delta - \overline{spr}) < 0. \tag{25}$$

Finally, let us notice that the value of any payment does not depend on the Ask price Q'_{Ask} noted in the moment of opening the order.

Additionally, let us assume that the investor when opening the transaction observes the current decision-making premise $s_j \in \mathbb{D}(Trh)$ characterised by a conditional probability $p(\mathcal{J}_\delta|s_j)$ of the currency exchange rate increase by a given magnitude of unitary return equal to δ.

If (14) is satisfied, then the investor places a BUY order. According to (21) and (25) the payment is then given as a random variable with the probability distribution:

$$\left\{\left(\mathscr{v} \cdot (\delta - \overline{spr}), p(\mathcal{J}_\delta|s_j)\right), \left(\mathscr{v} \cdot (-\delta - \overline{spr}), 1 - p(\mathcal{J}_\delta|s_j)\right)\right\}. \tag{26}$$

If (15) is satisfied, then the investor places a SELL order. According to (19) and (23) the payment is then given as a random variable with the probability distribution:

$$\left\{\left(\mathscr{v} \cdot (\delta - \overline{spr}), 1 - p(\mathcal{J}_\delta|s_j)\right), \left(\mathscr{v} \cdot (-\delta - \overline{spr}), p(\mathcal{J}_\delta|s_j)\right)\right\}. \tag{27}$$

All the above considerations indicate that the individual payments gained thanks to the DPT system use for one unit of base currency BCR, depend only on the value of spread $\overline{spr}$ and the magnitude of unitary return δ.

Additionally, opening the order depends on exceeding the decision-making threshold $\overline{Thr}$. To illustrate those facts, a DPT system which initiates – after exceeding the decision-making threshold $\overline{Thr}$ - the transaction with a magnitude of unitary return δ and bearing the spread $\overline{spr}$ will be denoted as $DPT(Thr, \delta, \overline{spr})$, where the values of δ and $\overline{spr}$ should be given in pips.

Example 2: In ex ample 1 a prediction table was presented for AUD/NZD foreign currency pair on the condition that the magnitude of unitary return $\delta = 20\,pips = 0.0020$. In the following part of the exemplary considerations, a constant spread $\overline{spr} = 1.4\,pips = 0.00014$ will be assumed. It is an average value of the spread offered by one of the more popular brokers ICMarkets. In this situation, all following examples will be carried out for the pair of AUD/NZD using a $DPT(\overline{Thr}, 20, 1.4)$ trading system. The subject of the consideration will be the transaction which equal 1 AUD lot. □

Example 3: Let the decision-making premise be given as a s_6 state. The probability of a currency exchange rate growth is then given by $p(\mathcal{J}_\delta|s_6) = 0.6979$. Then a BUY order is placed and according to (26) the probability distribution of payments is as follows $\{(186\,NZD;\ 0.6979), (-214\,NZD; 0.3021)\}$. □

Example 4: Let the decision-making premise be given by the state s_4. The probability of a currency exchange rate growth is then given by $p(\mathcal{J}_\delta|s_4) = 0{,}3492$. Then a SELL order is placed and according to (27) the probability distribution of payments is as follows $\{(186\,NZD;\ 0.6508), (-214\,NZD; 0.3492)\}$. □

To enable a more straightforward analysis, closing the order with a positive payment will be considered a success. According to (21), (23), (26) and (27) the probability $\pi(\mathcal{J}_\delta|s_j)$ of a successful finishing the transaction realised due to an observation of a current premise s_j equals:

$$\pi(\mathcal{J}_\delta|s_j) = \max\{p(\mathcal{J}_\delta|s_j), 1 - p(\mathcal{J}_\delta|s_j)\}. \tag{28}$$

The probability $\pi(\mathcal{J}_\delta|s_j)$ will be called the probability of success. The probability of success enables to assess the expected payment on the entered transactions due to the indications of a given prediction table. According to (26), (27) and (28) the probability of payment gained after the $DPT(\overline{Thr}, \delta, \overline{spr})$ closes any transaction equals:

$$\left\{\left(\mathscr{v} \cdot (\delta - \overline{spr}), \pi(\mathcal{J}_\delta|s_j)\right), \left(\mathscr{v} \cdot (-\delta - \overline{spr}), 1 - \pi(\mathcal{J}_\delta|s_j)\right)\right\}. \tag{29}$$

A positive expected payment is identified with an expected profit. Let us establish a minimal probability of success $\pi(\mathcal{I}_\delta|s_j)$ which guarantees that $DPT(\overline{Thr},\delta,\overline{spr})$ trading system will ensure an expected profit. This condition is presented by a following strict inequality:

$$\pi(\mathcal{I}_\delta|s_j)\cdot(\delta-\overline{spr})+\left(1-\pi(\mathcal{I}_\delta|s_j)\right)\cdot(-\delta-\overline{spr})>0. \tag{30}$$

After elementary transformations we get:

$$\pi(\mathcal{I}_\delta|s_j)>\frac{\delta+\overline{spr}}{2\cdot\delta}=\pi_{up}, \tag{31}$$

where the symbol π_{up} means a lower range of the success probability value. Obviously, any decision-making threshold $\overline{Thr}$ should satisfy the following condition:

$$\overline{Thr}\geq\pi_{up}. \tag{32}$$

Example 5: According to the prediction table described in example 1, any trading system $DPT(\overline{Thr},20{,}1.4)$ gives transactional recommendations given in table 2. It shows the probabilities of any closed successful transaction.

Tab. 2 Recommendations given by a trading system $DPT(\overline{Thr},20{,}1.4)$

State s_j	Recommendation	Probability of success $\pi(\mathcal{I}_\delta\|s_j)$
s_1	BUY	0.5733
s_2	BUY	0.5818
s_3	SELL	0.6047
s_4	SELL	0.6508
s_5	BUY	0.5441
s_6	BUY	0.6979
s_7	SELL	0.6137
s_8	SELL	0.5222

Source: own calculations. □

Example 6: For any $DPT(\overline{Thr};20;1{,}4)$ system the lower range of probability of success equals $\pi_{up}=0.5350$. Then the level of decision-making threshold is limited to $\overline{Thr}\geq0.5350$. According to a conditional probability of success presented in table 2, the state s_8 cannot be an acceptable decision-making premise. □

5. Criteria of the DPT trading system assessment

An important element of each decision-making process in any market is the proper choice of a trading system. The realization of that choice should be accompanied by a set of criteria selected in advance. In case of any $DPT(\overline{Thr},\delta,\overline{spr})$ trading system we can list the following criteria:

- The expected annual number of occurrence of the acceptable decision-making premise,
- The systemic probability of success,
- The expected single individual payment,
- The expected annual individual payment,
- Assessment of transactional risk,
- The individual single bonus due to risk,
- The annual single bonus due to risk.

The above listed criteria refer to the proposal presented in García et al. (2012) and Li et al. (2015).

The expected annual number $\mathcal{N}(\overline{Thr})$ of the acceptable decision-making premise occurrence can be presented as follows:

$$\mathcal{N}(\overline{Trh})=\sum_{s_j\in\mathbb{D}(\overline{Trh})}\tilde{n}_j, \tag{33}$$

where $\tilde{n}_j$ is a n_j number of annually calculated observations of the state s_j collected during constructing the prediction table.

The systemic probability of success $\pi\left(\mathcal{I}_\delta\middle|\mathbb{D}(\overline{Thr})\right)$ can be presented as:

$$\pi\left(\mathcal{J}_\delta \middle| \mathbb{D}(\overline{Thr})\right) = \frac{\sum_{s_j \in \mathbb{D}(\overline{Tr\square})} p(s_j) \cdot \pi(\mathcal{J}_\delta | s_j)}{\sum_{s_j \in \mathbb{D}(\overline{Tr\square})} p(s_j)}. \tag{34}$$

The expected single individual payment $\mathscr{y}(\overline{Thr}, \delta, \overline{spr})$ due to using a trading system equals:

$$\mathscr{y}(\overline{Thr}, \delta, \overline{spr}) = \pi(\mathcal{J}_\delta | \mathbb{D}(\overline{T\square r})) \cdot (\delta - \overline{spr}) + \left(1 - \pi(\mathcal{J}_\delta | \mathbb{D}(\overline{T\square r}))\right) \cdot (-\delta - \overline{spr}). \tag{35}$$

The expected annual single payment $\mathcal{Y}(\overline{\mathrm{Thr}}, \delta, \overline{\mathrm{spr}})$ due to using a trading system is:

$$\mathcal{Y}(\overline{Thr}, \delta, \overline{spr}) = \mathcal{N}(\overline{Trh}) \cdot \mathscr{y}(\overline{Thr}, \delta, \overline{spr}). \tag{36}$$

Criteria (33), (34), (35) and (36) represent the evaluation of $DPT(\overline{Thr}, \delta, \overline{spr})$ trading system. Along with the increase of the criteria level, the efficiency of the $DPT(\overline{Thr}, \delta, \overline{spr})$trading system grows. Criteria (33) and (34) are independent of market conditions represented by the spread. It is the drawback of those criteria. Criteria (35) and (36) which represent the most comprehensive assessment of the benefits of $DPT(\overline{Thr}, \delta, \overline{spr})$ system exploitation, are devoid of those disadvantages. Those criteria can be used by the investors to maximize their benefits.

To evaluate the transactional risk, we use an Expected Shannon's Entropy (1948). In the presented case this risk estimation $\mathcal{E}(\overline{Thr}, \delta, \overline{spr})$ is calculated as:

$$\mathcal{E}(\overline{Thr}, \delta, \overline{spr}) = \frac{-\sum_{s_j \in \mathbb{D}(\overline{Tr\square})} p(s_j) \cdot \left(\pi(\mathcal{J}_\delta | s_j) \cdot \ln \pi(\mathcal{J}_\delta | s_j) + \left(1 - \pi(\mathcal{J}_\delta | s_j)\right) \cdot \ln\left(1 - \pi(\mathcal{J}_\delta | s_j)\right)\right)}{\ln 2 \cdot \sum_{s_j \in \mathbb{D}(\overline{Tr\square})} p(s_j)}. \tag{37}$$

The efficiency of $DPT(\overline{Thr}, \delta, \overline{spr})$ trading system increases along with the decrease of (37) criterion. Summing up, the efficiency of each $DPT(\overline{Thr}, \delta, \overline{spr})$ trading system should be evaluated by a pair of:

$$\left(\mathscr{y}(\overline{Thr}, \delta, \overline{spr}), \mathcal{E}(\overline{Thr}, \delta, \overline{spr})\right) \tag{38}$$

or a pair of:

$$\left(\mathcal{Y}(\overline{Thr}, \delta, \overline{spr}), \mathcal{E}(\overline{Thr}, \delta, \overline{spr})\right). \tag{39}$$

A pair (38) enables to evaluate the efficiency of a single use of a $DPT(\overline{Thr}, \delta, \overline{spr})$trading system. While, a pair (39) should be used to evaluate a multiple use of the same trading system.

When assessing $DPT(\overline{Thr}, \delta, \overline{spr})$ trading system it is also worth to follow the criterion of a maximum individual single bonus $\mathscr{b}(\overline{Thr}, \delta, \overline{spr})$ for the risk, expressed as shown below:

$$\mathscr{b}(\overline{Thr}, \delta, \overline{spr}) = \frac{\mathscr{y}(\overline{T\square r}, \delta, \overline{spr})}{\mathcal{E}(\overline{T\square r}, \delta, \overline{spr})}. \tag{40}$$

To assess a $DPT(\overline{Thr}, \delta, \overline{spr})$trading system one can also use a criterion of a maximum individual annual bonus $\mathcal{B}(\overline{Thr}, \delta, \overline{spr})$ given by:

$$\mathcal{B}(\overline{Thr}, \delta, \overline{spr}) = \frac{\mathcal{Y}(\overline{Thr}, \delta, \overline{spr})}{\mathcal{E}(\overline{Thr}, \delta, \overline{spr})}. \tag{41}$$

Criteria (34), (35). (37) and (40) serve the purpose of DPT trading system evaluation regarding a single transaction. A group of criteria (33), (34), (36), (37) and (41) plays another role. Those criteria are useful if assessing the transaction that are repeated a multiple number of times. However, on the other hand, a DPT system was designed as a HFT system. Due to that, the use of the other group of criteria seems to be the right approach to the task of the optimum DPT trading system choice.

However, the ultimate choice of the assessment criteria is up to the end-users (investors) of the DPT systems.

Example 7. Let us compare two transactional systems: $DPT(0.5350,20,1.4)$ with a minimum transactional threshold and $DPT(0.6000,20,1.4)$ with a threshold increased by the investor. The estimated characteristics are presented in table 3.

Tab. 3 Juxtaposition of characteristics of trading systems $DPT(0.5350,20,1.4)$ i $DPT(0.6000,20,1.4)$

Characteristics	$DPT(0.5350,20,1.4)$	$DPT(0.6000,20,1.4)$
A set of acceptable decision-making premises $\mathbb{D}(\overline{Thr})$	$\{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$	$\{s_3, s_4, s_6, s_7\}$
An expected, annual number of the acceptable decision-making premise occurrence $\mathcal{N}(\overline{Trh})$	3074.5	1703.5
Systemic probability of success $\pi\left(\mathcal{J}_\delta \middle| \mathbb{D}(\overline{Thr})\right)$	0.5996	0.6317
An expected single individual payment $\mathcal{y}(\overline{Thr}, \delta, \overline{spr})$	25.85 NZD	38.66 NZD
An expected single annual payment $\mathcal{Y}(\overline{Thr}, \delta, \overline{spr})$	79 468.92 NZD	65 808.28 NZD
An assessment of a transactional risk $\mathcal{E}(\overline{Thr}, \delta, \overline{spr})$	0.9650	0.9457
A individual single risk premium $\mathcal{b}(\overline{Thr}, \delta, \overline{spr})$	26.79 NZD	40.88 NZD
An individual annual risk premium $\mathcal{B}(\overline{Thr}, \delta, \overline{spr})$	82 354.84 NZD	68 639.24 NZD

Source: Own calculations.

Drawing the conclusions from the characteristics in table 3 the authors eave to the readers. □

6. Conclusions

A DPT trading system proposed in the paper is a formalized model of transactional systems presented in (Stasiak, 2016, 2017a, 2017b; Stasiak and Wójcicka, 2018). Thanks to that formalization it is fully adapted to use the information collected via binary transformation of currency rate. Due to that the process of making a transactional decision lacks the influence of a white noise which appears in foreign exchange markets (Logue, and Sweeney, 1977). An important advantage of a DPT system is a relatively low numeric complexity of an automatic algorithm making the transactional decisions which is vital due to the fact that all the calculations must be executed in real time.

The elaborated set of evaluation criteria of a DPT system can be useful to choose optimum parameters of the implemented system on a particular foreign exchange market. On a particular market of a currency exchange pair characterized by a value of spread = $\overline{spr}$ one chooses the rest of the parameters of a $DPT(\cdot,\cdot,\overline{spr})$ trading system.

From the technical point of view the markets of particular currency pairs differ from one other when it comes to the spread and the number of transactions. For that reason, the choice of an optimum foreign currency exchange market can be made by indicating the optimum spread value for a given $DPT(\overline{Thr}, \delta, \cdot)$ trading system. The implemented method of the prediction table estimation also influences the efficiency of a transactional trading system. To assess the efficiency of an implemented method one can use the presented criteria $DPT(0.5, \delta, 0)$.

On the other side, proposed evaluation criteria are independent of invested quotes. Therefore, they cannot be applied for evaluation of transactions strategies.

All those possibilities of the implementation of the proposed trading system assessment criteria constitute the added value of that paper.

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