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THE MANAGEMENT OF CATASTROPHE INSURANCE RISK IN A STOCHASTIC APPROACH

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Abstract: *The article develops the structure of CAT bonds and their pricing. The use of financial instruments from the capital market is aimed at managing the process of catastrophic risk securitization. The model of pricing the CAT bond considered in this article takes into account stochastic interest rates and more generic catastrophe loss processes. In the sequence of two cases considered in the article the prices of CAT bonds with the default risk are estimated. The intensity of influence of moral hazard increases together with the intensity of a catastrophic event, variability of loss and also the interest rate risk of the insurer assets and it decreases along with the level of launching bond payment and the initial value of the insurer capital.*
In addition to the considerations related to the behavior of CAT bond prices, the basis (systematic) risk is also considered. The model considered in this article may be viewed as a general way of assessing the default-risk. The exemplary applications of this model are presented here. Structural restrictions in this model link the bond price to basic characteristics of assets, liabilities, and interest rates. This allows one to value bonds with unique features through the use of numerical analysis.

Key words: *CAT bonds, contingent claim, moral hazard, basis risk*

JEL codes: *E420, G240, M11, M41*

1. Introduction

The article refers to the insurance instruments, including moral hazard and basis risk, considered in the literature. This is aimed at the management of the insurance risk of a catastrophic type. The basis for the presented considerations are the financial instruments the application of which, in the process of managing the insurance company, aims at the securitization of the catastrophe risk. This means that they are aimed at financing the results of catastrophe incidents. The CAT bonds will be the financing tools in the case considered in this article. These bonds have a similar character as contingent claim capital CC, but in real conditions they are instruments of the financial market; they are catastrophe-linked bonds.

The goal of presented considerations is the formulation of a contingent claim model to price bonds issued directly by the insurer, by SPV company dependent on the main insurer when there is no default risk for claims initiated by catastrophe incident. The anticipation of results of possible catastrophes with big aggregated losses, expressed indirectly in specifically constructed indices of losses together with considering them in the terms and conditions of issued bonds, is the anticipation of their price. Therefore, the proper pricing of bonds is an element of insurance operations management for there is total or partial depreciation of the face value of the CAT bond, which influences the finance of the insurance company. In the model concerning the dynamics of value of insurance company assets significant assumptions will be adopted for interest rate risk and credit risk that, in a specific way, affect all other forms of risk. It will be explained in a more detailed manner in the part related to specification of assumptions of the assets value model. It should be taken into account that the presented empirical examples may accede to Polish reality concerning the manifestation of catastrophe risk, even though the capital market itself has not worked out proper projects of instruments which can be applied in the securitization of such a risk. National literature has shown the attempt to price a catastrophe bond in the aspect of the so-called two-factor function of the investor's usefulness, whose payee would be municipal authorities.

A specific project of CAT bonds has a historical context. In retrospect, one could observe a constant evolution of CAT designs and, consequently, also their pricing. Even though we can price default-risky CAT bonds, it is also possible to consider default-free CAT bonds prices (Szkutnik, 2009). We will also assume that there is moral hazard, which is associated with the event of claims pricing (the event that is inadequate to losses) that leads to losses either of the insurer (the most frequent case) or the bondholders and insurance policies holders. It should be remembered that catastrophe risk is partially insured in a traditional way by entities that are subject to such a risk and this procedure involves also insurance companies, typically reinsurance companies. We also consider basis risk, which is a symptom of the influence of capital market condition as well as of market risk.

2. Structural aspect of CAT bonds and trends in pricing CAT bond designs with the consideration of moral hazard and basis risk

In Polish literature the above mentioned catastrophe bonds were considered as debt instruments, burdened with the risk of the payee's insolvency. In Polish conditions the project of such instruments was addressed to self-government authorities in order to secure the region against the results of floods, droughts, forest fire etc. The structure of such bonds and also their efficiency consists in the fact that in case of catastrophe incident, which stimulates beforehand the issue of such instruments and generates financial losses and claims, the issuer is not obliged to pay back the nominal value of the bond and to continue the interest payment until they expire. However, when no catastrophe incident took place the bond holder (investor) is the beneficiary of such a favorable arrangement. Among them the most important are those that trigger the risk of debt, potential moral threat and "behavior" or the basic risk of the issuer.

Moral hazard can increase the claim payments at the expense of the investors' principal reduction and affect the bond price. The CAT bond's basis risk refers to the gap between the insurer's actual loss and the composite index of losses that prevents the insurer from receiving complete risk hedging.

The effect of moral hazard and basis risk was originally studied not from the perspective of the CAT bonds pricing but from the point of view of the influence of these types of risk upon the hedging process as a form of financing economic enterprises with various provenience, uncertain, yet burdened with significant losses in case of these enterprises' failure. The potentially new, restricted catastrophe-linked securities, related to the securitization of the effects of catastrophic events were also created. Bouzouita and Young (1998) focused on the regulations of the specific actions and applications from the perspective of risk management.

Among others, Cummins and Geman (1995) and Chang and Yu (1996) focused on the pricing of CAT futures and CAT call spreads under the condition of deterministic interest rate and specific property claims services (PCS) loss processes. There was also the pricing of one-year zero-coupon CAT bond which was further compared to the CAT bond price estimated by hypothetical catastrophe loss distribution. Zajdenweber (1998) followed), but he changed the catastrophe loss distribution to the stable Levy distribution. Louberge et al. (1991) numerically estimated the CAT bond price under the assumptions that the catastrophe loss follows a pure Poisson process, the individual losses have independently identical lognormal distribution, and the interest rate model is a binomial random process.

All the above listed pricing elaborations failed to incorporate a commonly acceptable stochastic interest rate process and catastrophe loss process as well as the default risk of the CAT bonds.

The article by Jin-Ping Lee and Min-The Yu (2002) develops a contingent claims model to price default-risky catastrophe-linked bonds, where interest rates have a stochastic character. Moreover, it allows for more generic loss processes and practical considerations of moral hazard, basis risk and default risk. There are estimations of both default-free and default-risky CAT bond prices. The results show that both moral hazard and basis risk drive down the bond prices substantially; therefore, these results should not be ignored in pricing the CAT bonds. There are also shown relations between the bond prices and the scale of claims caused by the catastrophe incident, loss volatility, and trigger level, dependent on the loss index, the issuing company's capital position, debt structure and interest rate uncertainty. The elaboration on this subject is important also from the practical point of view. This results from the fact that under accepted assumptions in the offered models of assets value, interest rate, loss model, the priced CAT bond hedge enable the issuer to avoid the credit risk that may arise with traditional reinsurance or catastrophe-linked options, which has already been mentioned.

As far as moral hazard is concerned, it should be remembered that it is initiated by the insurers themselves. This is related to the insurer's cost of loss pricing. Sometimes this cost exceeds the issuer's (SPV company) profits (insurer) that result from the debt value (the issuer) which occurs at the time of catastrophe incident. This stems from the specific character of CAT bonds. This means that the insurer has an incentive to pay the claims more generously when the loss amount is near the trigger set in the debt-forgiveness provision. Doherty (1997) pointed out that moral hazard results from less loss and control effort by the insurer issuing CAT bonds, because these efforts will increase the amount of debt that must be repaid. It is also noticed the tendency for insurers to write additional policies in a catastrophe-prone area, spending less time and money in their auditing of losses after a disaster. The effect of moral hazard may increase the claim payments at the expense of the bondholders' principal reduction and affect the bond price.

Another important aspect, which must be considered in pricing a CAT bond, is the basis risk. As it is already known, the basis risk of CAT bond refers to the gap between the insurer's actual loss and the composite index of losses that prevents the insurer from receiving complete risk hedging. The basis risk may cause insurers to default on their debt in the case of high individual loss but low index of loss, and therefore affects the bond price. However, there exists a balance between the basis risk and moral hazard. If one uses an independently calculated index to define the CAT bonds payments, then the insurer's opportunity to cheat the bondholders is reduced or eliminated. This is equal to a lesser scope of moral hazard behavior or even its elimination. However, the basis risk is created.

3. Aggregate loss dynamics

Following the typical setting for loss dynamics in the actuarial the **aggregate loss model** can be expressed as a compound Poisson process, a sum of jumps.

To estimate the impact of basis risk on the CAT bond price the term $C_{i,t}$ that denotes the aggregate loss for the issuing company i is introduced; the term $C_{index,t}$ represents that for a composite index of losses (e.g., a PCS index). These two processes can be described as follows:

$$C_{i,t} = \sum_{j=1}^{N(t)} X_{i,j}, \tag{1}$$

and

$$C_{index,t} \sum_{j=1}^{N(t)} X_{index,j}, \tag{2}$$

where the process $\{N(t)\}_{t\geq 0}$ is the loss number process described by Poisson process with intensity λ. Symbols $X_{i,j}$ and $X_{index,j}$ denote the Mount of losses caused by the jth catastrophe during the specific period for the issuing insurance company and the composite index of losses, respectively. It is assumed that terms $X_{i,j}$ and ($X_{index,j}$), for j = 1, 2, …, $N(T)$, are mutually independent and have identical lognormal distribution, and they are also independent of the loss number process, and their logarithmic means and variances are denoted by μ_i (μ_{index}) and σ_i^2 (σ_{index}^2), respectively. In addition, assume that ρ correlation coefficients of the logarithms of $X_{i,j}$ and ($X_{index,j}$), for different j = 1, 2, . . . , N (T) are identical.

3.1 Loss dynamics under the risk-neutralized pricing measure

To carry out the pricing of CAT bonds one needs to know the loss dynamics under the risk neutralized pricing measure Q. When the loss process has sudden jumps, **the market is called then called incomplete** and there is no unique pricing measure.

Thus, follow Merton (1976) and assume that the **economic conditions are only marginally influenced** by localized catastrophes such as earthquakes and hurricanes, and that the loss number process $\{N(t)\}$ and the amount of losses $X_{i,j}$ ($X_{index,j}$), are directly related to idiosyncratic **'shocks'** to the capital markets, it means **the factors that influence the capital markets in an inexpedient way**.

These factors- catastrophic shocks represent 'nonsystematic' risk and have a zero risk premium, which they generate.

By assuming that such a jump risk is nonsystematic and diversifiable, attaching a risk premium to the risk is unnecessary. It turns out that this assumption is important because one cannot apply a risk-neutral evaluation to situations in which the size of the jump is systematic. This point is minutely discussed by Naik and Lee (1990), Cox and Pedersen (2000).

Therefore, it can be accepted that the aggregate loss processes expressed by equations and retain their original distributional characteristics after changing from the physical probability measure to the risk-neutralized pricing measure.

3.2 The influence of basis risk and moral hazard on the price and payment of CAT bonds

It turns out that once the risk-neutral process of asset dynamics, loss and interest rate are known it is possible to estimate the CAT bond price by the discounted expectation of its various payoffs in the risk-neutral world. The specification of payoffs of the CAT bond may be carried out under alternative considerations concerning the payoff risk. In this aspect, we can consider first the basic case, in which there is no default risk and then also the case of the default-risky CAT bonds with potential basis risk and moral hazard.

To price the CAT bond it is assumed that this is a hypothetical discount bond whose payoffs (PO_T) at maturity (i.e., time T) are as follows:

$$PO_T = \begin{cases} a \cdot L & \text{if } C_T \leq K \\ r \cdot p \cdot a \cdot L & \text{if } C_T > K \end{cases} \tag{3}$$

where: K – trigger levels the bond redemption established in the provisions of CAT bond bonds;
C_T – total loss on payment date;
$r \times p$ – part of the capital that must be paid to the bond holders when the redemption was launched;
L – nominal value of the issuer's total debts, of which the nominal value of CAT bonds is part;
a – ratio of the nominal sum of CAT bonds to total outstanding debts.

4. Approximation of the aggregate loss distribution and the bond price – analytical solution

Under the assumption that the catastrophe loss amount components are independent and identically lognormally distributed the exact distribution of the aggregate loss at maturity date, denoted as $f(C_T)$, is obviously not known in the exact form. However, an approximate analytical form of this probability distribution can be set up. For this purpose, we approximate the exact distribution by a lognormal distribution, denoted as $g(C_T)$, with specified moments. Nielson and Sandmann (1996) used the same assumptions in approximating the values of Asian options and the so-called basket options.

In this case, we can consider different options, for example when determining default risky withdrawals, when there is no underlying risk or when it occurs.

4.1 Determination of payoffs with consideration of moral hazard

The third of the possible versions of pricing default-risky bonds concerns the structure of modeling payoffs with the moral hazard.

We may encounter such a situation when the CAT bond forgiven on the issuing company's own losses because the issuing company has the priority and an incentive to settle claims, which has been already mentioned, more "generously" when the loss incurred approaches the trigger level. This model assumes that the issuing company "relaxes" its settlement policy once the accumulated losses fall into the range close to the trigger. This assumption is justified with the fact the accumulation of losses would cause an increase in expected losses for the catastrophic event. In this case the change of the loss process shall be specified as follows:

$$\mu_i' = \begin{cases} (1+\alpha) \cdot \mu_i, & \text{if } (1-\beta) \cdot K \leq C_{i,j} \leq K, \\ \mu_i & \text{otherwise,} \end{cases} \tag{4}$$

where μ' is a logarithmic mean of the losses incurred by the (j +1) th catastrophe when the accumulated loss $C_{i,j}$ falls in the specified range, $(1–\beta) K \leq C_{i,j} \leq K$; α is a positive constant, reflecting the percentage increase in the mean; and β is a positive constant that specifies the range of moral hazard behavior or the solution, which does not have to be fully specified and which is introduced *ad hoc* for the purpose of determining the approach towards moral hazard when it is necessary but without prior assumption of complex conditionings that led to its occurrence. In the aspect of the last assumption concerning constant β that modifies the change of loss process it should be marked that literature does not settle this aspect of moral hazard and, therefore, it should be rather regarded as a challenge that inspires other scientists to model moral hazard.

The presented analytical structure of the complex contingent contract is the basis for its complementation and modification. However, presented in the current form, it does not allow for numerical estimation of the CAT bond price. It has already been signaled that this will be the subject of the exemplification of the application of CAT bond pricing model on the basis of the data simulated by the Monte Carlo method.

4.2 The exemplification of the model structure of CAT bonds with the application of Monte Carlo simulation

For the purpose of explaining a bit difficult, from the formal point of view, analytical structure of the CAT bond estimation, we will carry out the exemplification of application of model structure for this bond (Lee and Yu, 2002; Szkutnik, 2009). The CAT bond prices were estimated (Lee and Yu, 2002) by the Monte Carlo simulation. We will consider the pricing of default-free bonds. We will not take into account the default-risky bonds with moral hazard and basis risk.

The initial step in pricing the CAT bond is the established set of parameters and base values. To assess the comparative effects of these parameters on CAT bond prices deviations from the base values are also established. For simplicity, it is assumed that the total amount of the issuing company's debts, which include CAT bonds, has a face value of \$100 and that the maturity of the CAT bond is equal to one year. The simulations are run on a weekly basis with 20,000 paths. The given parameters ad base values are presented in table 1.

Tab. 1 Parameters and the base values

Parameter	Description	Values
A/L	insurer assets to liabilities:	1.1, 1.2 or 1.3
μ_A	drift due to credit risk	
Φ	interest rate elasticity of asset	0, -3, -5
σ_A	volatility of credit risk	5%
W_A	Wiener process for credit shock	

Source: Own elaboration.

The initial asset/liability (or capital) position (*A/L*) ratios are set to be 1.1, 1.2, and 1.3, respectively. The average *A/L* ratio for the insurance sector equaled about 1.3 on a book-value basis over the past ten years. The interest rate elasticity of the insurer's assets is set at 0, -3, and -5, respectively to measure how the insurer's interest rate risk affects CAT bond prices. The volatility of the asset return that is caused by the credit risk is set at the level of 5%.

Table 2 includes the interest rate parameters. The initial spot interest rate *r* and the long-run interest rate *m* are both set at 5%. The magnitude of mean-reverting force⌛ is set to be 0.2, while the volatility of the interest rate *v* is set at 10%. The market prices λ_r of interest rate are set at 0 and -0.01, respectively. All these term structure parameters are included within the range typically used in the previous literature.

Tab. 2 Interest rate parameters

Parameter	Description	Values
r	initial instantaneous interest rate	5%
⌛	magnitude of mean-reverting force	0.2
m	long-run mean of interest rate	5%
v	volatility of interest rate	10%
λ_r	market prices of interest rate risk	0, -0.01
Z	Wiener process of interest rate shock	

Source: Own elaboration.

Table 3 presents the catastrophe loss parameters and other parameters, including the trigger levels for debt-forgiveness and the ratio of principal needed to be paid if debt forgiveness is triggered.

Tab. 3 Catastrophe loss parameters

Parameter	Description	Values
	Catastrophe loss parameters	
μ_i	mean of the logarithm of the amount of catastrophe losses for the insurer	2
μ_{indeks}	mean of the logarithm of the amount of catastrophe losses for the composite loss index	0.2
σ_i	standard deviation of the logarithm of the amount of catastrophe losses for the insurer	0.5, 1, 2
σ_{indeks}	standard deviation of the logarithm of the amount of catastrophe losses for the composite loss index	0.5, 1, 2
ρ_x	correlation coefficient of the logarithms of amounts of catastrophe losses of the insurer and the composite loss index	0.5, 0.8, 1
$N(t)$	Poisson process for the occurrence of catastrophes	
	Other parameters	
K	trigger levels	100, 110, 120
Rp	the ratio of principal needed to be paid if debt forgiveness has been triggered	0.5
a	the ratio of the amount of CAT bond to total debts	0.1
α	moral hazard intensity	20%
β	the ratio set below the trigger that will cause the insurer's moral hazard	20%
L	the total amount of insurer's debts	100

Source: Own elaboration.

The occurrence intensities of catastrophe losses are set to be 0.5, 1, and 2, respectively, to reflect the frequencies of catastrophic incidents per year. Also assume that the parameter values for catastrophe loss are the same for individual insurers and the composite loss index. We set the logarithmic mean μ_i and μ_{indeks} to be 2, and the logarithmic standard deviations, σ_i and σ_{indeks} to be 0.5, 1 and 2. The values for the index and individual insurers can be differently modified, but it increases the numerical dimension of calculations and it does not broaden the analysis of basis risk. The analysis will focus on the coefficient of correlation ρ_x between the individual loss and the loss index rather than on their means and standard deviations. The portion of principal needed to be repaid, $r{\cdot}p$, is set at 0.5 when debt forgiveness has been triggered. The ratio of the amount of CAT bonds to the insurer's total debt a, is set at 0.1. Additionally, there are three different trigger levels K set at 100, 110, and 120.

Table 4 includes the prices of default bonds with the alternative occurrence and non-occurrence of moral hazard at the alternative values of initial capital position (*A/L*), catastrophe intensity, loss variance and the interest rate elasticity of the issuing company's assets. Three (upper, middle, lower) values reported in each cell of table 4 represent the corresponding estimate under the interest rate elasticity of 0, -3 and -5, respectively. A higher (absolute) value of interest rate elasticity corresponds to higher asset volatility and default risk of the issuer. Thus, one would expect the upper value of each cell (the CAT bond price for $\phi =$ 0) to be higher than the middle value (the CAT bond price for $\phi =$ -3) and lower value (the CAT bond price for $\phi =$ -5).

It is also expected that the higher the initial capital position (*A/L*) of the issuing company is, the lower the default risk and the higher the CAT bond prices are. The rise of the bond price is caused by the occurrence intensity and catastrophe loss volatility. All estimated were computed using 20 000 simulation runs. Bond prices are per face amount of one dollar. The upper value, middle value, and lower value in each cell are CAT bond prices computed when the interest rate elasticities of the asset are 0, -3, -5, respectively.

Tab. 4 Prices of default–risky CAT bonds with and without moral hazard (λ_r = - 0.01)

	With moral hazard* *Triggers K								
(λ,σ_i)	**100**	**110**	**120**	*100*	*110*	*120*	100	110	120
	0.72232	**0.72628**	**0.72693**	*0.72946*	*0.73576*	*0.75965*	0.73492	0.74182	0.74922
(2,2)	**0.72055**	**0.72410**	**0.72528**	*0.72882*	*0.73421*	*0.73756*	0.73554	0.74163	0.74629
	0.71928	**0.72253**	**0.72445**	*0.72757*	*0.73222*	*0.73310*	0.73355	0.74101	0.74445

Source: Simulated data – own elaboration

4.3 The numerical pricing of default-risky CAT bonds with the alternative consideration of moral hazard

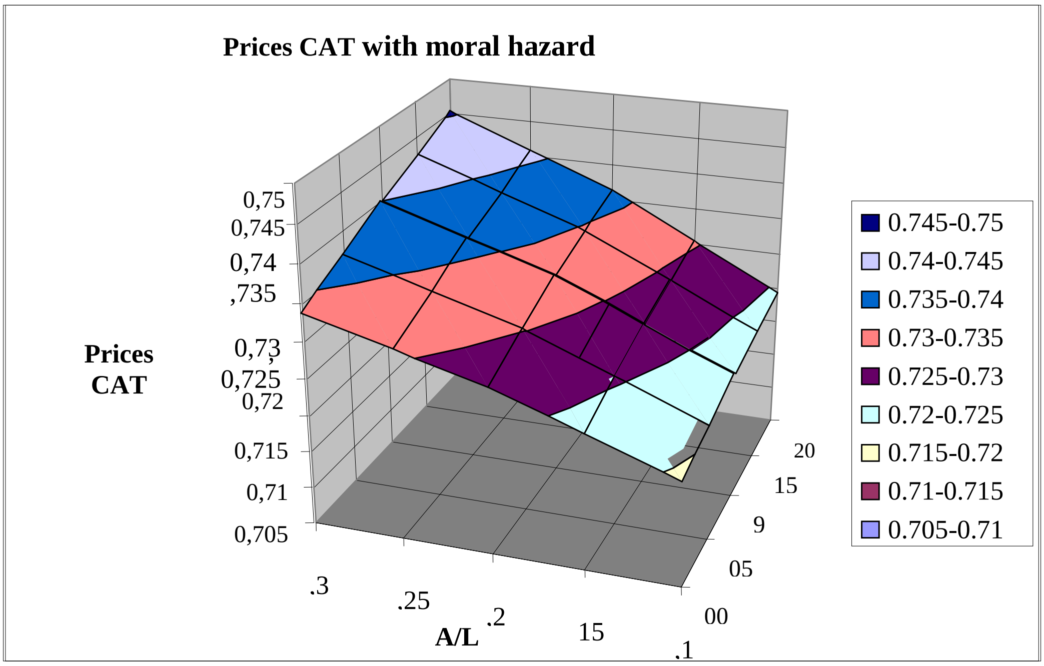
The prices of default bonds with the alternative occurrence and non-occurrence of moral hazard at the alternative values of initial capital position (*A/L*), catastrophe intensity, loss variance and the interest rate elasticity of the issuing company's assets are presented in table 4. Three (upper, middle, lower) values reported in each cell of the table represent the corresponding estimate under the interest rate elasticity of 0, -3 and -5, respectively. A higher (absolute) value of interest rate elasticity corresponds to higher asset volatility and default risk of the issuer. Thus, one would expect the upper value of each cell (the CAT bond price for $\phi =$ 0) to be higher than the middle value (the CAT bond price for $\phi =$ -3) and lower value (the CAT bond price for $\phi =$ -5).

It is also expected that the higher the initial capital position (*A/L*) of the issuing company is, the lower the default risk and the higher the CAT bond prices are. The rise of the bond price is caused by the occurrence intensity and catastrophe loss volatility. All estimated were computed using 20 000 simulation runs. Bond prices are per face amount of one dollar. The upper value, middle value, and lower value in each cell are CAT bond prices computed when the interest rate elasticities of the asset are 0, -3, -5, respectively.

Let us observe that the default risk premium decreases with the *A/L* ratio and increases with the occurrence intensity and loss volatility. The default risk premium can go as high as 1,015 basis points for the case of *A/L* = 1.1, $(\lambda, \sigma) = (2, 2)$, and $\phi = -5$.

To incorporate the effect of moral hazard it is assumed that when the accumulated los amounts to 80% of the trigger level, the insurer will settle the catastrophe claims more generously and therefore will increase the expected loss of the catastrophe by 20%, which is reflected by the coefficient of moral hazard intensity $\alpha = 0.2$. The moral hazard, therefore, increases the default risk and lower the bond price. For example, in the case of $(\lambda, \sigma) = (2,2)$, $\phi = -5$, the price decreases about 350 basis points with the moral hazard. The magnitude of the moral hazard effect increases with (λ, σ_i) and absolute value of ϕ, and decreases with the *A/L* ratio.

Fig. 1 Prices of default-risky CAT bonds with moral hazard (λ, σ) = (2, 2), and ϕ = -3



Source: Own elaboration.

Figure 2 presents the price of default-risky CAT bonds with moral hazard and parameters of intensity, volatility and elasticity: (λ, σ) = (2,2), and ϕ = -5, at various trigger levels of *A/L* bond payoff. The triggers of 1.1, 1.15, 1.2, 1.3 are marked on the axis of abscissae. We also observe that CAT bond price increases with the increase of the trigger. This increase of bond price is, as previously noted, also influenced by the increase of intensityλ, and loss volatility rate σ_i.

The above figure reflects the case when loss volatility is constant (σ_i = 2) and asset/liability ratio *A/L*= 1.1. The illustration from figure 2 suggests that in extreme cases, when loss intensity is low (level of *l* = 0.5), irrespective of the trigger of bond payoff, the risky bond premium with moral hazard will reflect the payoff in connection with the incurred losses. The increase of loss intensity *l* to level 2 significantly decreases the bond payoff. The CAT bond prices illustrated with and without moral hazard under alternative values of parameters (*l*, σ_i) in figures 1 and 2, as well as in case that presents the CAT bond prices at variable trigger levels and variable loss intensity and with moral hazard, show the clear dependence of these prices on basis risk measured by *A/L* ratio and on the loss intensity *l*. These figures show, at the same time, a slight influence of trigger level at a constant basis risk *A/L* on the CAT bond price. The significant price differences indicate that the moral hazard is an important factor ad should be taken into consideration when pricing the CAT bonds.

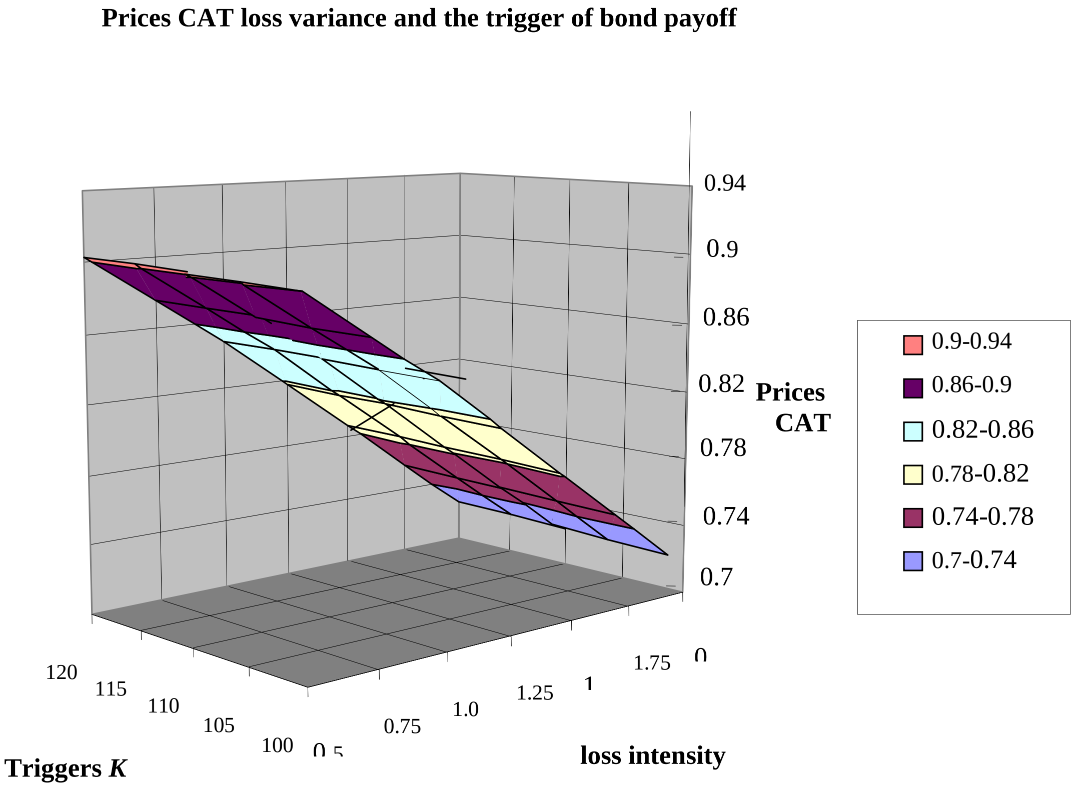
5. Conclusions

In the article considered account stochastic interest rates and more generic catastrophe loss processes takes into the model of pricing the CAT bond. It is also possible to ‘measure’ the impacts of default-risk, moral hazard, and basis risk that are associated with CAT bonds. In case of no default risk it is stated that the CAT bond prices computed numerically are very close to the ones computed by the approximating solution, except that when the loss volatility is high. Then, the approximated prices reach higher values.

The presented analytical structure of the complex contingent contract is the basis for its complementation and modification. However, presented in the current form, it does not allow for numerical estimation of the CAT bond price. It has already been signaled that this will be the subject of the exemplification of the application of CAT bond pricing model on the basis of the data simulated by the Monte Carlo method.

For the two cases considered in the article the prices of CAT bonds with the default risk are estimated. There is also the analysis of premium in the conditions of default risk that is changing along with the amount of claims caused by a given catastrophic event, variability of loss, flexibility of interest rate of the insurer assets, the coefficient of initial capital and the structure of liabilities. The premium is estimated also with regard to the influence of moral hazard upon prices (value) of CAT bonds. In this context, it is stated that moral hazard significantly lowers the value of bonds. The intensity of influence of moral hazard increases together with the intensity of a catastrophic event, variability of loss and also the interest rate risk of the insurer assets and it decreases along with the level of launching bond payment and the initial value of the insurer capital.

Fig. 2 Prices of default-risky CAT bonds with $A/L = 1.1$ with moral hazard and $\phi = -5$ and constant volatility



Source: Own elaboration.

In addition to the considerations related to the behavior of CAT bond prices, the basis (systematic) risk is also considered. Basis risk significantly decreases the prices of CAT bonds and it contributes to the decrease of the rate of these prices. The influence of basis risk increases along with the trigger level, intensity of a catastrophic event, variability of loss and it decreases along with the initial value of the capital.

The model considered in this article may be viewed as a general way of assessing the default-risk. The exemplary applications of this model are presented here. Structural restrictions in this model link the bond price to basic characteristics of assets, liabilities, and interest rates. This allows one to value bonds with unique features through the use of numerical analysis. It is important to note that this model can be easily extended to analyze other default-risky liabilities, not only these concerning CAT bonds, but also insurance-linked securities.

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