HOW TO INCLUDE EXPERTS' IMPRECISION IN CREDIT RISK ASSESSMENT?

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Abstract: *Credit risk assessment usually is a complex process which consists of many successive steps and numerous criteria. Selection of good customers and rejection of potentially bad ones is vital as it directly and significantly affects the quality of bank's credit portfolio. It is often the experts' responsibility to make the final decision, but their knowledge, experience and expertise often take a form of real natural language which is inaccurate an imprecise. That imprecision results from the use of common language expressions like more/less, higher/lower or bigger/smaller. Even though those expressions (labels) are difficult to quantify, they frequently influence the final decision.*

Also, those labels are often used to order the decision alternatives as it is also an important part of the whole decision-making analysis which takes place before making a final decision. The importance and complexity of the problem on one hand call for strictly analytical methods, however, on the other, also for a method which enables intuitive decision-making, imprecision and inaccurate linguistic ranks based on experts' personal experience.

The main objective of the paper is to capture the significance of imprecisely expressed professional opinion of experts, and in turn, utilizing their knowledge, experience and preferences in the process of making a credit risk decision. This is achieved by the implementation of Simple Additive Weighting (SAW) method (multicriteria decision-making method), the linguistic approach and the order scale described by ordered fuzzy numbers (OFN).

Key words: *credit risk assessment, SAW method, OFN*

JEL codes: *C02, C35, sC38, G40*

1. Introduction

The complexity of credit risk evaluation processes requires running it in separated steps. Those steps usually cover a scrutinized analysis of borrowers' financial statements (usually of at least three successive years). Basing on that analysis a borrower is given a rating and a recommendation follows. At that point, from a risk management perspective, transparency throughout the process is crucial and it allows the bank to see an accurate and comparable picture. However, this is also at that stage of the process that measurable and quantitative analysis stops. For the decision is taken at the meeting of a credit assessment committee, consisting of experts (higher level managers) who make their decisions basing on their personal, professional experience. Their main role is to use their ability to look at the future projections of the borrower's business and not just their past performance. Any underwriting agreements, financial projections and the health of the borrower's industry are all very important, as they will be leading indicators of potential volatility in loan payments, however, assessed features can frequently be conflicting or excluding one another. The final decision can be consistent with the recommendation or it can reject the recommendation.

As the final decision frequently bases on inaccurate and imprecise, linguistic premises, the potential borrowers may be evaluated by means of a rating scale utilising their preferences. Those preferences are expressed by weights given to each group of criteria. The analysed criteria are the features which influence the final decision.

Therefore, the main objective of the paper is to capture the significance of imprecisely expressed professional opinion of experts, and in turn, utilizing their knowledge, experience and preferences in the process of making a credit risk decision. This is achieved by the implementation of Simple Additive Weighting (SAW) method (multicriteria decision-making method), the linguistic approach and the order scale described by ordered fuzzy numbers (OFN). SAW method can deal with qualitative dimensions and its utilization of provides experts with a support system allowing them to reach final decision based on linguistic, imprecise criteria. Also, Oriented Fuzzy SAW (OF-SAW) method (Piasecki, Roszkowska, 2018) can be applied for solving the problem to assess the standing of the potential debtor. The proposed system might integrate fuzzy set theory and the SAW method to evaluate the available alternatives.

The paper is organized as follows. Section 2 presents the overview of credit risk assessment methods. Section 3 outlines the basics on Ordered Fuzzy Numbers (OFNs) which is followed by description of general linguistic approach to the borrowers' evaluation and rankings produced on the linguistic approach method and ordered scale (in Section 4). Section 5 sets the grounds for Simple Additive Weighting method. Section 6 presents the numerical example, which illustrates the procedure of the proposed SAW method and contributes into the understanding of the process of borrowers' evaluation. Finally, Section 7 concludes the article, summarizes the main findings of this research and proposes some future research directions.

2. Credit risk assessment methods – overview

The Basel Capital Accord within the Internal Rating Based Approach (IRB) allows banks to use their own, chosen rating models for the estimation of probabilities of default (PD). Banks can use various approaches to classify the borrowers. The methods, which are most widely used in credit risk assessment and the evaluation of borrowers, usually belong to a group of parametric methods such as linear discriminant analysis, regression analysis, credit scoring and non-parametric methods such as neural networks, expert systems or support vector machines, machine learning etc. All these methods have their limitations. Amongst those limitations we can enlist: misclassification and indirect discrimination, variations from market to market, problems with accommodating changes, assuming a specific normality or homoscedasticity that are often violated in real world or model selection on trial and error process (Grace and Williams, 2016).

Moreover, it is important to remember that in the field of credit risk assessment research, we can also include, not just the bank's debtors but also other creditors (e.g. trading companies) that play a role of creditor in trade transactions with a delayed payment (trade credit). Using that tool, one party is always threatened by credit risk. Therefore, for them intuitive systems of debtors' assessment such as OF-SAW are really essential. Another approach is the statistical enterprise trade credit risk assessment model for evaluation of trade risk of small and micro enterprises (Kanapickiene and Spicas, 2019).

Within the area of credit risk assessment and debtors' ratings there are also many modifications and extensions[1]. They appear as a result of the shortcomings of existing models. The significance of experts' knowledge and experience, as well as other qualitative factors in credit risk assessment and debtors' classification, are recognised as increasingly influential and helpful in decision-making process. In (Grace and Williams, 2016) neural network and fuzzy logic systems for credit risk evaluation was developed and their performances were evaluated

[1] A review on financial risk assessment (including credit and bankruptcy risks) can be found in Chen et al. (2016)

based on prediction accuracy metric. The conclusion was that despite comparable results, the fuzzy inference system could be easily understood by any user, however, the decisions made by the neural network system is not easily understood by the user, and in this case the user has no choice than to accept the output given by the neural network as the most appropriate output without any explicit reasoning. Also, in (Dadios & Solis, 2012) a hybrid fuzzy logic and neural network algorithm (HFNN) to solve credit risk management problem is tested. It is shown that HFNN model can solve credit risk management problem and is capable of self- learning similar to the traditional neural network. It can also generate the rules behind the discrimination of each account subjected to it and in this manner, it behaves much like a traditional fuzzy logic system.

3. Oriented Fuzzy Numbers – Basic Facts

Objects of any considerations may be given as elements of a predefined space $\mathbb{X}$. The basic tool for an imprecise classification of these elements is the notion of fuzzy sets introduced by Zadeh (1967). Any fuzzy set $\mathcal{A}$ is unambiguously determined by means of its membership function $\mu_A \in [0,1]^{\mathbb{X}}$, as follows

$$\mathcal{A} = \{(x, \mu_A(x)); x \in \mathbb{X}\}. \tag{1}$$

From the point-view of multi-valued logic (Łukasiewicz, 1922/23), the value $\mu_A(x)$ is int erpreted as the truth value of the sentence "$x \in \mathcal{A}$". By the symbol $\mathcal{F}(\mathbb{X})$ the family of all fuzzy sets in the space $\mathbb{X}$ is denoted.

Dubois and Prade (1979) have introduced fuzzy numbers (FNs) as such a fuzzy subset in the real line which may be interpreted as imprecise approximation of a real number. The ordered FNs were intuitively introduced by Kosiński et al. (2002) as an extension of the FNs concept. Ordered FNs usefulness follows from the fact that it is interpreted as FNs with additional information about the location of the approximated number. Currently, ordered FNs defined by Kosiński are often called Kosiński's numbers (Piasecki, 2019). A significant drawback of Kosiński's theory is that there exist such Kosiński's numbers which, in fact, are not FNs (Kosiński 2006). For this reason, the Kosiński's theory was revised by Piasecki (2018). If an ordered FN is determined with use of the revised definition, then it is called Oriented FN (OFN). The OFN definition fully corresponds to the intuitive Kosiński's definition of ordered FNs.

In this paper, he analysis is restricted to the case of Trapezoidal OFNs (TrOFN) defined as fuzzy subsets in the space $\mathbb{R}$ of all real numbers in the following way.

Definition 1: (Piasecki, 2018) For any monotonic sequence $(a,b,c,d) \subset \mathbb{R}$, TrOFN $\overleftrightarrow{Tr}(a,b,c,d) = \overleftrightarrow{T}$ is the pair of the orientation $\overrightarrow{a,d} = (a,d)$ and a fuzzy subset $\mathcal{T} \in \mathcal{F}(\mathbb{R})$ determined explicitly by its membership functions $\mu_T \in [0,1]^{\mathbb{R}}$ as follows

$$\boldsymbol{\mu_T(x) = \mu_{Tr}(x|a,b,c,d)} = \begin{cases} \mathbf{0}, & \boldsymbol{x \notin [\min\{a,d\}, \max\{a,d\}]}, \\ \frac{x-a}{b-a}, & \boldsymbol{x \in [\min\{a,b\}, \max\{a,b\}[}, \\ \mathbf{1}, & \boldsymbol{x \in [\min\{b,c\}, \max\{b,c\}]}, \\ \frac{x-d}{c-d}, & \boldsymbol{x \in]\min\{c,d\}, \max\{c,d\}]}. \end{cases} \tag{2}$$

The symbol $\mathbb{K}_{Tr}$ denotes the space of all TrOFNs. Any TrOFN describes an imprecise number with additional information about the location of the approximated number. This information is given as orientation of OFN. If $a < d$ then TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ has the positive orientation $\overrightarrow{a,d}$. For any $z \in [b,c]$, the positively oriented TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ is a formal model of linguistic variable "about or slightly above z". If $a > d$, then OFN $\overleftrightarrow{Tr}(a,b,c,d)$ has the negative orientation $\overrightarrow{a,d}$. For any $z \in [c,b]$, the negatively oriented TrOFN $\overleftrightarrow{Tr}(a,b,c,d)$ is a formal model of linguistic variable "about or slightly below z". Understanding the phrases "about or slightly above z" and "about or slightly below z" depends on the applied pragmatics of the natural language. If $a = d$, then TrOFN $\overleftrightarrow{Tr}(a,a,a,a) = [\![a]\!]$ describes un-oriented real number $a \in \mathbb{R}$.

Kosiński has introduced the arithmetic operators of dot product $\odot$ for TrOFNs in a following way:

$$\beta \odot \overleftrightarrow{Tr}(a,b,c,d) = \overleftrightarrow{Tr}(\beta \cdot a, \beta \cdot b, \beta \cdot c, \beta \cdot d). \tag{3}$$

In Piasecki (2018), the sum $\boxplus$ for TrOFNs is determined as follows

$$\overleftrightarrow{Tr}(a,b,c,d) \boxplus \overleftrightarrow{Tr}(p-a, q-b, r-c, s-d) .= \begin{cases} \overleftrightarrow{Tr}(\min\{p,q\}, q, r, \max\{r,s\}) & (q < r) \vee (q = r \wedge p \le s) \\ \overleftrightarrow{Tr}(\max\{p,q\}, q, r, \min\{r,s\}) & (q > r) \vee (q = r \wedge p > s) \end{cases} \tag{4}$$

Let us consider the pair $(\overleftrightarrow{\mathcal{K}}, \overleftrightarrow{\mathcal{L}}) \in \mathbb{K}_{Tr}^2$ represented by the pair $(\mu_K, \mu_L) \in ([0,1]^{\mathbb{R}})^2$ of their membership functions. On the space $\mathbb{K}_{Tr}$, the relation $\overleftrightarrow{\mathcal{K}}.\widetilde{GE}.\overleftrightarrow{\mathcal{L}}$, is introduced, which reads:

$$\text{"}TrOFN\ \overleftrightarrow{\mathcal{K}}\ is\ greater\ than\ or\ equal\ to\ TrOFN\ \overleftrightarrow{\mathcal{L}}.\text{"}. \tag{5}$$

This relation is a fuzzy preorder $\widetilde{GE} \in \mathcal{F}(\mathbb{K}_{Tr}^2)$ defined by its membership function $\nu_{GE} \in [0,1]^{\mathbb{K}_{Tr}^2}$ (Piasecki, 2019; Piasecki et al, 2019). From the point of view of the multivalued logic, the value $\nu_{GE}(\overleftrightarrow{\mathcal{K}}, \overleftrightarrow{\mathcal{L}})$ is considered as a truth-value of the sentence (5). In (Piasecki, 2019), it is shown that for any pair $(Tr(a,b,c,d), Tr(e,f,g,h)) \in \mathbb{K}_{Tr}^2$ we have

$$\nu_{GE}\left(\overleftrightarrow{Tr}(a,b,c,d),\overleftrightarrow{Tr}(e,f,g,h)\right) =$$

$$\begin{cases} 0, & 0 < \alpha - \gamma, \\ \frac{\alpha-\gamma}{\alpha+\delta-\beta-\gamma}, & \alpha - \gamma \leq 0 < \beta - \delta,. \\ 1, & \beta - \delta \leq 0 \end{cases} \tag{6}$$

where

$$\alpha = \max\{a, d\} \tag{7}$$

$$\beta = \max\{b, c\} \tag{8}$$

$$\gamma = \min\{e, h\} \tag{9}$$

$$\delta = \min\{f, g\} \tag{10}$$

Therefore, for any pair $(\boldsymbol{Tr}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}), [\![\boldsymbol{e}]\!]) \in \mathbb{K}_{\boldsymbol{Tr}} \times \mathbb{R} \subset \mathbb{K}_{\boldsymbol{Tr}}^{\mathbf{2}}$ we get

$$\nu_{GE}\left(\overleftrightarrow{Tr}(a,b,c,d), [\![e]\!]\right) =.$$

$$= \begin{cases} 0, & max\{a,d\} < e, \\ \dfrac{max\{a,d\} - e}{max\{a,d\} - max\ \{b,c\}}, & max\{a,d\} \geq e > max\ \{b,c\}\ , \\ 1, & 0 \leq c - f. \end{cases} \tag{11}$$

4. Linguistic Approach – Order Scales

Credit granting is a decision which is tainted by credit risk understood as the possibility that credit will default. Credit lenders tend to minimize this risk. For this reason, they evaluate borrowers in terms of many criteria.

Any borrower attributes can be evaluated by means of numerical values. By its very nature of things, each such assessment is an imprecise information. Therefore, in dealing with such a situation with imprecise information, the use of linguistic assessments, instead of numerical values, may be more useful. Following (Herrera, Herrera-Viedma, 2000), it can be said that an application of imprecise linguistic assessments for decision analysis is very beneficial because it introduces a more flexible framework which allows us to represent the information in a more direct and adequate way when it is difficult or impossible to express it precisely. However, by means of ranking systems, the qualitative concept can be translated into a quantitative one.

In the first step of any linguistic approach, the imprecision granularity should be determined, i.e., the cardinality of the linguistic term set used for showing the information. The imprecision granularity indicates the capacity of distinction that may be expressed. The knowledge value is increasing with the increase in granularity. The typical values of cardinality used in the linguistic models are odd ones, usually between 3 and 13. It is worth to note that the idea of granular computing goes from Zadeh (1997) who wrote ‘‘fuzzy information granulation underlies the remarkable human ability to make rational decisions in an environment of imprecision, partial knowledge, partial certainty and partial truth.’’ Also, Yao (2004) pointed out that ‘‘the consideration of granularity is motivated by the practical needs for simplification, clarity, low cost, approximation …’’. For review variety of application linguistic models in decision-making see for example (Herrera, Herrera-Viedma, 2000).

In general (Herrera, Herrera-Viedma, 2000), any linguistic value is characterized by means of a label with semantic value. The label is an expression belonging to a given linguistic term set. Finally, a mechanism of generating the linguistic descriptors is provided.

In credit risk assessment, all linguistic assessments are linked with Tentative Order Scale (TOS) given as a sequence

$$TOS = \{Bad\ , Average\ , Good\ \} = \{C, B, A\} = \{V_1, V_2, V_3\}. \tag{12}$$

Any element of TOS is called a reference point and can be enlarged by intermediate values. For this purpose, the following orientation phrases can be used:

- “much below” described by the symbol $" - -$“,
- “below” described by the symbol $" - "$,
- “around” described by the symbol $"\sim"$,
- “above” described by the symbol $" + "$,
- “much above” described by the symbol $" + +"$.

Any order label is determined as a composition of reference point and orientation phrases. The set of all order labels is called Extended Order Scale (EOS). In Table 1, TOS and EOS proposed for credit risk assessment are presented.

In information sciences, natural language word is considered as a linguistic variable defined as a fuzzy subset in the predefined space $\mathbb{X}$. Then, these linguistic variables may be transformed with the use of fuzzy set theory (Zadeh, 1975). From decision making point view, the linguistic variable transformation methodologies are reviewed in (Herrera et al, 2009).

Let us assume that each reference point V_j is represented by the number $j \in \mathbb{N}$. On the other side, the semantic meaning of any orientation phrase is imprecise. For this reason, any order label may be considered as imprecise approximation of its reference point. Thus, each order label from applied EOS should be represented in the real line $\mathbb{R}$ by FN (Chen, Hwang, 1992). For convenience of future calculations, this representation can always be restricted to representation by trapezoidal FN. Moreover, the observation is made that orientation phrases determine the orientation of FN representing approximated reference point. Therefore, any order label can be represented by TrOFN. This approach is more faithful than representation of order labels by trapezoidal FN. On the other hand, an omission of information about order labels' orientation causes unbelievable assessment of borrowers (Piasecki et al, 2019 b). For these reasons, all order labels will be represented by TrOFNs. The family of all TrOFNs representing considered EOS will be called Numerical Order Scale (NOS). In credit risk assessment task, NOS is used. All applied order scales are presented in Table 1.

Tab. 1 Order scale

TOS	EOS	Semantic meaning	NOS
	C--	much below Bad	$\overleftrightarrow{Tr}\left(1, 1, \frac{3}{4}, \frac{1}{4}\right)$
	C-	below Bad	$\overleftrightarrow{Tr}\left(\frac{5}{4}, 1, \frac{3}{4}, \frac{2}{4}\right)$
	C~	around Bad	$\overleftrightarrow{Tr}\left(\frac{2}{4}, 1, 1, \frac{6}{4}\right)$
C		Bad	$\overleftrightarrow{Tr}(1, 1, 1, 1)$
	C+	above Bad	$\overleftrightarrow{Tr}\left(\frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}\right)$
	C++	much above Bad	$\overleftrightarrow{Tr}\left(1, 1, \frac{5}{4}, \frac{7}{4}\right)$
	B--	much below Average	$\overleftrightarrow{Tr}\left(2, 2, \frac{7}{4}, \frac{5}{4}\right)$
	B-	below Average	$\overleftrightarrow{Tr}\left(\frac{9}{4}, 2, \frac{7}{4}, \frac{6}{4}\right)$
	B~	around Average	$\overleftrightarrow{Tr}\left(\frac{6}{4}, 2, 2, \frac{10}{4}\right)$
B		Average	$\overleftrightarrow{Tr}(2, 2, 2, 2)$
	B+	above Average	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$
	B++	much above Average	$\overleftrightarrow{Tr}\left(2, 2, \frac{9}{4}, \frac{11}{4}\right)$
	A--	much below Good	$\overleftrightarrow{Tr}\left(3, 3, \frac{11}{4}, \frac{9}{4}\right)$
	A-	below Good	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
	A~	around Good	$\overleftrightarrow{Tr}\left(\frac{10}{4}, 3, 3, \frac{14}{4}\right)$
A		Good	$\overleftrightarrow{Tr}(3, 3, 3, 3)$
	A+	above Good	$\overleftrightarrow{Tr}\left(\frac{11}{4}, 3, \frac{13}{4}, \frac{14}{4}\right)$
	A++	much above Good	$\overleftrightarrow{Tr}\left(3, 3, \frac{13}{4}, \frac{15}{4}\right)$

Source: own calculations

5. Simple Additive Weighting Method - Overview

Let us take into account the problem of borrower's evaluation. The evaluation template distinguishes all borrower's attributes which are evaluated. Any borrower may be evaluated by means of a scoring function which takes into account experts' preferences with respect to all evaluation criteria and their relative importance. The process of determining evaluation template is an important part of credit risk analysis, as well as constructing a scoring function, which is realized in the pre-evaluation phase. Because borrowers are often characterized by several contradictory criteria, the multi-criteria techniques are useful for building borrower-scoring function. The most popular techniques used for multi-criteria evaluation is the Simple Additive Weighting (SAW) method (Mardani et al, 2015). The SAW method is a scoring method based on the concept of a weighted average of criterion ratings. In the considered task of a credit risk evaluation, the individual criterion ratings are expressed by TrOFNs. For this reason, SAW method linked with TrOFNs is needed. Such SAW method should be equipped with scoring function determined on the space $\mathbb{K}_{Tr}^{n} = \mathbb{K} \times \mathbb{K} \times \ldots \times \mathbb{K}$.

The SAW method is also called Simple Multi Attribute Rating Technique. In (Piasecki, Roszkowska, 2018) Oriented Fuzzy SAW (OF-SAW) method is modified in a way that it is compatible with the revised theory of ordered FNs (Piasecki, 2018). In this case, criterion ratings are given as TrOFNs. Below, the OF-SAW method is adapted to the needs of assessing a single borrower.

The intention is to evaluate a borrower characterized by attributes record $\mathcal{A} \in \mathbb{A}$ where $\mathbb{A}$ is an anticipated set of potential borrowers. For this case OF-SAW method can be described by the following procedure:

Step 1: Define a multi-criteria evaluation problem by criteria set $\mathbb{D} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\}$.

Step 2: Determine the weight vector

$$w = (w_1, w_2, \ldots, w_n) \in (\mathbb{R}_0^+)^n \tag{13}$$

where

$$w_1 + w_2 + \cdots + w_n = 1. \tag{14}$$

and w_j is the weight of the criterion $\mathcal{C}_j$ denoting the importance of this criterion in considered evaluation problem

Step 3: For each evaluation $\mathcal{C}_j$ $(j = 1,2, \ldots, n)$, determine its scope Y_j.

Step 4: Determine the evaluation template

$$\mathbb{Y} = Y_1 \times Y_2 \times \ldots . \times Y_n \supset \mathbb{A}. \tag{15}$$

Step 5: Define the NOS $\mathbb{O} \subset \mathbb{K}_{tr}$.

Step 6: Define the evaluation function $\mathcal{X}: \mathbb{Y} \times \mathbb{D} \rightarrow \mathbb{O} \subset \mathbb{K}_{tr}$ in such way that the value $\mathcal{X}(\mathcal{A}, \mathcal{C}_j) \in \mathbb{O}$ is equal to evaluation of attributes record $\mathcal{A}$ from the point-view of the criterion $\mathcal{C}_j$ $(j = 1,2, \ldots, n)$.

Step 7: Determine the scoring function $\overleftrightarrow{SAW}: \mathbb{Y} \rightarrow \mathbb{K}_{Tr}$ given for any $\mathcal{A} \in \mathbb{Y}$ by the identity

$$\overleftrightarrow{SAW}(\mathcal{A}) = \\ = (w_1 \odot \mathcal{X}(\mathcal{A}, \mathcal{C}_1)) \boxplus (w_2 \odot \mathcal{X}(\mathcal{A}, \mathcal{C}_2)) \boxplus \ldots \boxplus (w_n \odot \mathcal{X}(\mathcal{A}, \mathcal{C}_n)). \tag{16}$$

For a given evaluation template $\mathbb{Y}$, any classical scoring method of credit risk assessment can be presented as a pair (f, L) (Mays, 2001; Anderson, 2007) where:

$f: \mathbb{Y} \longrightarrow \mathbb{R}$ is a given scoring function,

$L \in \mathbb{R}$ is a predetermined level of acceptance of a credit/loan application.

Let us consider a credit application of a borrower characterized by attributes record $\mathcal{A} \in \mathbb{A}$. If the following condition is fulfilled

$$f(\mathcal{A}) \geq L. \tag{17}$$

then the application is acceptable (Mays, 2001; Anderson, 2007).

In this section to assess the creditworthiness it is suggested to use a scoring function $\overleftrightarrow{SAW}: \mathbb{Y} \rightarrow \mathbb{K}_{Tr}$. Therefore, it is also suggested to extend the inequality (16) into a following form

$$\overleftrightarrow{SAW}(\mathcal{A}).\widetilde{GE}.[\![L]\!]. \tag{18}$$

The fulfilment of the above inequality is tantamount to a sentence:

$$Credit\ application\ based\ on\ attributes\ record\ \mathcal{A}\ is\ acceptable. \tag{19}$$

Then the value $\nu_{GE}(\overleftrightarrow{SAW}(\mathcal{A}), [\![L]\!])$ is truth-value of the sentence (19). For this reason, we interpret the value $\nu_{GE}(\overleftrightarrow{SAW}(\mathcal{A}), [\![L]\!])$ as a degree in which the considered credit application is acceptable. Therefore, the value

$$accept(\mathcal{A}, L) = \nu_{GE}(\overleftrightarrow{SAW}(\mathcal{A}), [\![L]\!]). \tag{20}$$

will be called the acceptance degree (acceptance level). This value can be a significant premise for the credit committee to take a final decision to grant the funding.

6. Numerical example – case study

This paper has applied 16 criteria that are qualitative and positive for selecting a good potential debtor amongst the analysed ones and ranking them. The introduced method is used in a case study.

The data was collected from two experts in the banking field who are active members of a credit assessment committee with a long business experience in that field[2]. The research was conducted in the following steps:

Step 1. Preparation of an appropriate assessment form (template).

The most important part of that research stage was to establish the qualitative criteria, basing on experts' business experience in credit risk assessment. Eventually, the experts settled on 16 criteria which, apart from the quantitative analysis of financial ratios, influence the final decision. The chosen criteria are presented in Table 2.

Step 2. Incorporate weights of the criteria

In the research basing on the experts' professional knowledge the criteria were divided into 5 groups (see Table 2). Each group was given a ranking grade (a rank) of a real number ranging from 1 to 5 where 1 meant "the least important group" and 5 meant "the most important group". The weights of each group were calculated as a quotient of the given rank (respectively 1, 2, 3, 4 or 5) and the sum of the ranks (15). This way the least important group had the weight 1/15 and the most important group was given the weight of 5/15. The weights of the individual criterion within a given group were equal and ranged from ¼ to ½ depending on the number of criteria in the group. The incorporated weights are presented in Table 2 Groups of criteria are presented from the most important to the least important.

Tab. 2 Groups of chosen criteria with the ranks

rank of the group (1-5) / weight of the group	criteria / (weight I the group)	weights of an individual criterion (within the group)
5 / 0,333	risk of the market / (0,25) risk of the trade / (0,25) risk of the supplier / (0,25) risk of the customer / (0,25	0,083
4 / 0,267	diversification of the product / (0,333) diversification of the sale markets (0,333) diversification of the supply market (0,333)	0,089
3 / 0,2	prospects of business / (0,333) quality of suppliers / (0,333) quality of customers / (0,333)	0,067
2 / 0,133	clean criminal record of the Board members / (0,25)	0,033

[2] The personal data of experts and any data concerning the Bank as well as any business and decision-making actions involved in the process, are subject to confidentiality.

1 / 0,067	clean criminal record of a chairperson / (0,25) experience of the Board members / (0,25) experience of a chairperson / (0,25) operations range – Poland / (0,5) operations range – abroad / (0,5)	0,033

Source: own calculations

Step 3. Set the acceptance level.

Before the beginning of the evaluation process, the experts were informed that the acceptance level is represented by *“a middle point between reference points ‘Average’ and ‘Good’* ”. The experts evaluated credit application provided by ‘Enterprise A’ and ‘Enterprise B’.

Step 4. Experts fill in the form.

The experts express their individual, professional opinion on the specific criterion in relation to an analysed enterprise by attributing that criterion to a single rank of EOS.

Step 5. Transform the experts’ evaluations into NOS.

The evaluations given by each expert were transformed into NOS.

The obtained results show that the experts differ in their perception of the importance of the qualitative features when assessing the same entity. Therefore, as a final assessment, the mean SAW value representing common opinion of both experts was calculated (Table 3).

Tab. 3 Values of scoring function SAW

	Evaluations		
Enterprise	SAW by Expert 1	SAW by Expert 2	Mean SAW
A	$\overleftrightarrow{Tr}\left(\frac{70}{32},\frac{72}{32},\frac{75}{32},\frac{83}{32}\right)$	$\overleftrightarrow{Tr}\left(\frac{132}{64},\frac{132}{64},\frac{136}{64},\frac{146}{64}\right)$	$\overleftrightarrow{Tr}\left(\frac{137}{128},\frac{138}{128},\frac{143}{128},\frac{156}{64}\right)$
B	$\overleftrightarrow{Tr}\left(\frac{127}{64},\frac{136}{64},\frac{144}{64},\frac{156}{64}\right)$	$\overleftrightarrow{Tr}\left(\frac{150}{64},\frac{152}{64},\frac{159}{64},\frac{175}{64}\right)$	$\overleftrightarrow{Tr}\left(\frac{267}{128},\frac{288}{128},\frac{303}{128},\frac{331}{128}\right)$

Source: own calculations

In the next step, for each assessed value of SAW scoring function the acceptance degree values are calculated (20). Due to the fact that the acceptance level is represented by “a middle point between reference points ‘Average’ and ‘Good’“, it is assumed that the acceptance level is given as

$$L = \frac{1}{2} \cdot (2+3) = \frac{5}{2}. \tag{21}$$

Here, we utilise the relationship (11). All values acquired in this manner are presented in Table 4.

Tab. 4 Acceptance degree

	Acceptance degree		
	by Expert 1	by Expert 2	of Mean SAW
Enterprise A	0.425	0.000	0.000
Enterprise B	0.000	0.8750	0.4677

Source: own calculations

The values presented in Table 4 allow to formulate the following findings:

- credit application of Enterprise A is accepted at medium level by Expert 1;
- credit application of Enterprise A is not accepted by Expert 2;
- credit application of Enterprise A is not accepted by experts' team;
- credit application of Enterprise B is not accepted by Expert 1;
- credit application of Enterprise B is strongly accepted by Expert 2;
- credit application of Enterprise B is accepted at medium level by experts' team.

The final decision of granting the credit (loan) is up to the credit committee. The committee can take into consideration the opinions presented above.

7. Conclusions

Credit risk assessment usually is a complex process which consists of many successive steps and numerous criteria. The distinction between a good and a bad customer and following rejection of those in the latter group is vital for a bank as it directly and significantly affects the quality of bank's credit portfolio. The final decision in the process of credit decision-making is always tinted by experts' professional knowledge and preferences. Therefore, the nature of the problem enables intuitive decision-making, imprecision and inaccurate linguistic ranks based on experts' personal experience.

The calculations, conducted in a numerical example presented in the paper, show the utility of SAW method in case of a credit risk assessment and the order scale is described by oriented fuzzy numbers (OFN).

The estimation of the acceptance level and the individual weights of criteria within the group should be a subject of further research.

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